MAT 102 – Exam3 - Spring 2015

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I certify that I did not receive third party help in *completing* this test (sign) _____

Instructions. Each problem is worth 10 points. If you solve a problem graphically then draw the graph you used. Remember to check your solutions and "box" them reduced to lowest terms or with decimal numbers rounded to two decimal places. You might need some of the following formulas:

- $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
- $A^3 \pm B^3 = (A \pm B)(A^2 \mp AB + B^2)$
- $(A \pm B)^2 = A^2 \pm 2AB + B^2$
- $A^2 - B^2 = (A - B)(A + B)$

1. Solve the polynomial equation $x^3 - 14x^2 + 13x = 0$, by factoring or using the quadratic formula, making sure to identify all the solutions.

$X(X^2 - 14X + 13) = X(X-13)(X-1) = 0$

$X = 0$ $\quad X = 13$ $\quad X = 1$

2. Consider the following polynomial inequality.

$$(x - 2)(x + 5)(3 - x) \leq 0$$

Step 1. Set one side equal to zero and list the interval endpoints (the only points at which the non-zero expression can change sign). Separate multiple answers with a comma.

$X - 2 = 0 \rightarrow X = 2$

$X + 5 = 0 \rightarrow X = -5$

$3 - X = 0 \rightarrow X = 3$

Step 2. Test each interval to find which ones satisfy the inequality: you could also use the graph. **Write your answer in interval notation.**

		-5	2	3	
X	-6	0	2.5	4	
	(-)(-)(+)	(-)(+)(+)	(+)(+)(+)	(+)(+)(-)	
POLY	+	−	+	−	
≤		✓		✓	

$[-5, 2] \cup [3, \infty)$

3. Use polynomial long division to rewrite the following fraction in the form $q(x) + \frac{r(x)}{d(x)}$, where $d(x)$ is the denominator of the original fraction, $q(x)$ is the quotient, and $r(x)$ is the remainder.

$$\frac{8x^4 - 12x^3 + 20x^2 + 3x + 2}{2x^2 + 2} = \frac{f(x)}{2(x^2+1)}$$

$$\begin{array}{r|l} & 8x^2 - 12x + 12 \\ \hline x^2+1 & 8x^4 - 12x^3 + 20x^2 + 3x + 2 \\ & -8x^4 \qquad\quad - 8x^2 \\ \hline & -12x^3 + 12x^2 + 3x + 2 \\ & 12x^3 \qquad\quad + 12x \\ \hline & 12x^2 + 15x + 2 \\ & -12x^2 \qquad - 12 \\ \hline & 15x - 10 \end{array}$$

$$\longrightarrow = \frac{1}{2}\left(8x^2 - 12x + 12 + \frac{15x - 10}{x^2+1}\right)$$

$$= 4x^2 - 6x + 6 + \frac{5}{2}\,\frac{3x-2}{x^2+1}$$

4. Consider the factored polynomial

$$f(x) = (x + 2)(x - 1)^5.$$

Step 1. Determine the degree and y-intercept (write the y -intercept as an ordered pair).

DEGREE $= 1+5=6$; $2 \cdot (-1)^5 = -2 \rightarrow$ Y-INT. $(0, -2)$

Step 2. Determine the x-intercept(s) at which f crosses the axis. If there are none, state "none".

BOTH ODD MULTIPLICITY $\rightarrow$ $X = -2, 1$

X-INT: $(-2, 0)$ AND $(1, 0)$
CROSSING

Step 3. Determine the zero(s) of f at which it "flattens out". If there are none, state "none".

FLATTENS $\rightarrow$ MULTIPLICITY $\geq 2 \rightarrow$ $X = 1$ OR $(1, 0)$

5. Consider the polynomial function $f(x) = 3x^3 + 11x^2 + 8x - 4$.

Step 1. Factor the above polynomial function completely.

$f(-2) = 0$ AND BOUNCING $\rightarrow$ MULTIPLICITY 2 $\rightarrow$ $(X+2)^2$ DIVIDES $f(x)$

GRAPH

$f(x) = 3x^3 + 12x^2 - x^2 + 12x - 4x - 4$

$= 3x^3 + 12x^2 + 12x - x^2 - 4x - 4$

$= 3x(x^2 + 4x + 4) - (x^2 + 4x + 4) = (3x - 1)(x + 2)^2$

Step 2. Determine the degree and y-intercept (write the y -intercept as an ordered pair).

DEGREE $= 3$; Y-INTERCEPT $(0, -4)$

Step 3. Determine the x-intercept(s) at which f crosses the axis. If there are none, state "none".

ODD MULTIPLICITY $\rightarrow$ CROSSES AT $\left(\frac{1}{3}, 0\right)$

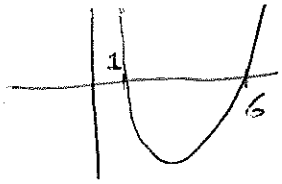
Step 4. Determine the zero(s) of f at which it "flattens out". If there are none, state "none".

FLATTENS $\rightarrow$ MULTIPLICITY $\geq 2 \rightarrow$ $(-2, 0)$

6. Use all available methods (in particular, the Conjugate Roots Theorem, if applicable) to factor the following polynomial function completely, making use of the given zero, $f(x) = x^4 - 7x^3 + 22x^2 - 112x + 96$; $-4i$ is a zero.

$-4i$ ZERO $\Rightarrow$ $+4i$ IS A ZERO $\Rightarrow$ $(x-(-4i))(x-4i)$ ARE FACTORS $\Rightarrow$

$\Rightarrow (x+4i)(x-4i) = (x^2+16)$ DIVIDES $f(x)$ $\Rightarrow$ LONG DIVISION OR MANIPULATE OR GRAPH



OTHER ZEROS $x = 1, 6$. THEN

$f(x) = (x+4i)(x-4i)(x-1)(x-6)$

CHECK Y-INTERCEPT: $16 \cdot (-1)(-6) = 96$ ✓

7. Construct a polynomial function with the following properties: fifth degree, 1 is a zero of multiplicity 3 , -2 is the only other zero, leading coefficient is 2. Sketch the graph.

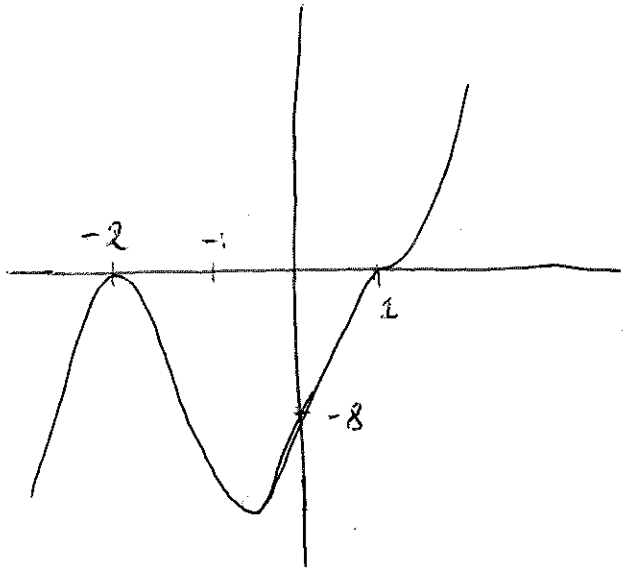
$f(x) = 2(x-1)^3(x-(-2))^2$ TO HAVE DEGREE $= 3+2 = 5$

2 $\rightarrow$ LEADING COEFF.

3 $\rightarrow$ REQUESTED

$f(x) = 2(x-1)^3(x+2)^2$

Y-INTERCEPT: $2(-1)^3(2)^2 = -8$



8. Given the following rational function:

$$f(x) = \frac{4x^2 - 4x - 15}{2x + 3}$$

Step 1. Find equations for the vertical asymptotes, if any, for the rational function.
Step 2. Find equations for the horizontal or oblique asymptotes, if any, for the rational function.

$$\begin{array}{r|l} 2X+3 & 2X-5 \\ & 4X^2-4X-15 \\ & -4X^2-6X \\ \hline & -10X-15 \\ & -10X-15 \\ \hline & 0 \end{array}$$

$f(x) = 2x-5 + \frac{0}{g(x) + \frac{r(x)}{d(x)}}$ ← NON-V.A.

OBLIQUE AS.: $y = 2x-5$

THERE IS NONE V.A. ← V.A.

BUT DOMAIN $x \neq -\frac{3}{2}$

9. Consider the following rational function:

$$f(x) = \frac{-6x^2 + 4x + 10}{-2x + 6} = \frac{-2(3x^2 - 2x - 5)}{-2(x-3)}$$

Find equations for the oblique asymptotes, if any, for the rational function.

$$\begin{array}{r|l} X-3 & 3X+7 \\ & 3X^2-2X-5 \\ & -3X^2+9X \\ \hline & 7X-5 \\ & -7X+21 \\ \hline & 16 \end{array}$$

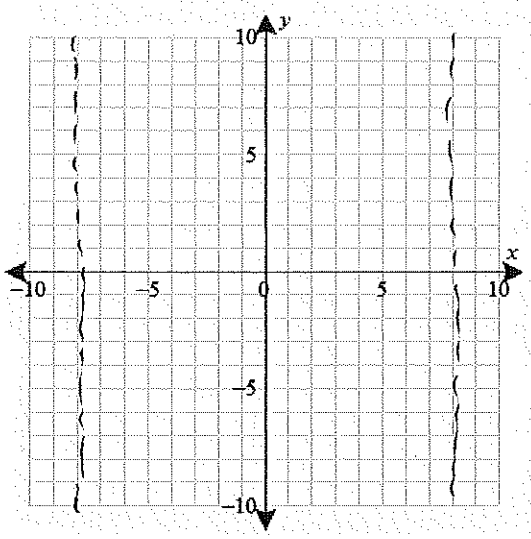
$f(x) = 3x+7 + \frac{16}{x-3}$

O.A.: $y = 3x+7$

10. Graph the following rational function.

$$f(x) = \frac{-x + 1}{x^2 - 64}$$

Step 1. Draw the vertical asymptotes, if any, of this function on the graph provided. How many vertical asymptotes does this function have?



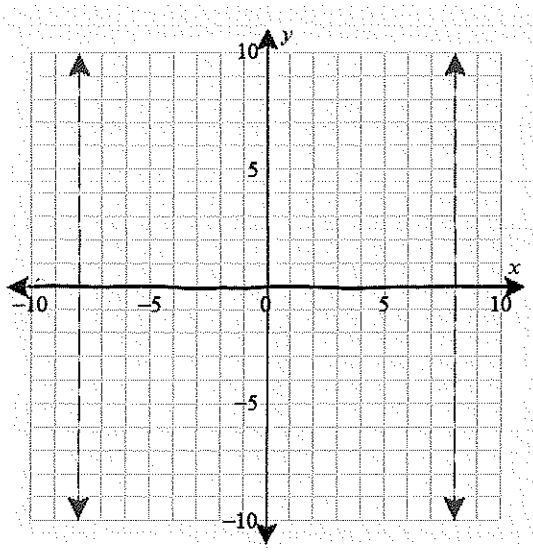
A) None B) One ~~C)~~ Two

$X^2 - 64 = 0 \Rightarrow X = \pm 8$

V.A. $X = 8$, $X = -8$

CHECK: $f(8) = \frac{*}{0}$ AND $f(-8) = \frac{*}{0}$ ✓

Step 2. Draw the horizontal asymptotes, if any, of this function on the graph provided. How many horizontal asymptotes does this function have?



A) None B) One C) Two

DEGREE NUM < DEGREE DEN $\Rightarrow$

$\Rightarrow y = 0$ HORIZ. ASYMP.

$f(x) = 0 + \frac{1 - X}{X^2 - 64}$

Step 3. Sketch the complete graph of the function.