MAT 121 – Exam3– Spring 2019– In Class MWF

Instructor: Dr. Francesco Strazzullo **Name**________________________________

Instructions. Complete the following exercises. Each exercise is worth 10 points. If you need to approximate then **round to 3 decimal places**, unless otherwise specified. This is an open book test. You can also use a graphing tool and/or a computer algebra system like GeoGebra. When solving a problem graphically sketch the graph you used.
SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

1. Determine the average, amplitude, period, and phase shift of the following trigonometric model.

$$f(x) = 4 - 2\mathrm{TAN}\left(\frac{\pi}{5}x + \frac{3\pi}{4}\right) = k + a \cdot \tan(bx + c)$$

If there is no phase shift, state "no phase shift". If there is a phase shift, state the direction of the phase shift and the number of units (as a positive number) the graph is shifted.

BASE PERIOD OF TANGENT $= \pi \Rightarrow p = \frac{\pi}{|b|} = \frac{\pi}{\pi/5} = 5$

PHASE SHIFT $= \left|\frac{c}{b}\right| = \frac{3\pi/4}{\pi/5} = \frac{15}{4}$ UNITS

$\frac{c}{b} > 0 \Rightarrow$ SHIFT TO LEFT.

2. Olivia just got a ride on ATL Ferris wheel. What are her linear and angular speeds if the diameter of the wheel is 215 feet and one "flight" is equal to FIVE revolutions, lasting about 20 minutes? Round your solutions to two decimal places.

FIVE REVOLUTUTIONS IN 20 MINUTES $\Rightarrow \theta = 5 \cdot 2\pi = 10\pi$ IN $T = 20 \Rightarrow$

$\Rightarrow \omega = \theta/T = \frac{10\pi}{20} = \frac{\pi}{2}$ RAD/MIN ANGULAR SPEED

≈ 1.57

$V = r\omega = \frac{215}{2} \cdot \frac{\pi}{2} = \frac{215}{4}\pi \approx 168.86$ FT/MIN

MAT 121 – Exam3– Spring 2019– In Class TR

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SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

1. Determine the average, amplitude, period, and phase shift of the following trigonometric model.

$$f(x) = 4 - 2\sin\left(\frac{\pi}{5}x + \frac{3\pi}{4}\right) = k + a\sin(bx + c)$$

If there is no phase shift, state "no phase shift". If there is a phase shift, state the direction of the phase shift and the number of units (as a positive number) the graph is shifted.

AVG $= k = k$; PERIOD $= \frac{2\pi}{|b|} = \frac{2\pi}{\pi/5} = 10$

APLITUDE $= |a| = 2$;

PHASE SHIFT $= \left|\frac{c}{b}\right| = \frac{3\pi/4}{\pi/5} = \frac{15}{4} = 3.75$

$\frac{c}{b} > 0 \Rightarrow$ SHIFT TO LEFT

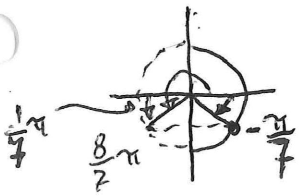
2. Olivia just got a ride on ATL Ferris wheel. What are her linear and angular speeds if the diameter of the wheel is 210 feet and one "flight" is equal to four revolutions, lasting about 22 minutes? Round your solutions to two decimal places.

FOUR REVOLUTIONS IN 22 MIN $\Rightarrow \theta = 4(2\pi) = 8\pi$ IN $T = 22 \Rightarrow$

$\Rightarrow$ ANGULAR SPEED $\omega = \frac{\theta}{T} = \frac{8\pi}{22} = \frac{4}{11}\pi \approx 1.14$ RAD/MIN

LINEAR SPEED : $v = r\omega = \frac{210}{2} \cdot \frac{4}{11}\pi = \frac{420}{11}\pi \approx 119.95$ FT/MIN

3. Simplify the following expression, without approximating:
$\sin^{-1}\left(\sin\left(\frac{8}{7}\pi\right)\right)$



$\frac{8}{7}\pi > \frac{\pi}{2}$. $\sin\left(\frac{8}{7}\pi\right) = \sin\left(\pi + \frac{\pi}{7}\right) = \sin\left(-\frac{\pi}{7}\right) \Rightarrow$

$\Rightarrow \sin^{-1}\left(\sin\left(\frac{8}{7}\pi\right)\right) = \sin^{-1}\left(\sin\left(-\frac{\pi}{7}\right)\right) = -\frac{\pi}{7}$

(DO NOT APPROX. TO $-.4488$)

4. Solve the following logarithmic equation. If needed, round your answer to 4 decimal places.

$\log_5(x+3) + \log_5(2x-3) = 1 \Rightarrow \log_5((x+3)(2x-3)) = 1 \overset{\text{EXPO.}}{\Rightarrow} 5^{LHS} = 5^{RHS}$

$\Rightarrow (x+3)(2x-3) = 5^1 \Rightarrow 2x^2 + 3x - 9 = 5 \Rightarrow$

$\Rightarrow 2x^2 + 3x - 14 = 0 \underset{3}{\overset{-28}{\Rightarrow}} 2x^2 + 7x - 4x - 14 = 0 \Rightarrow$

$\Rightarrow x(2x+7) - 2(2x+7) = 0 \Rightarrow (2x+7)(x-2) = 0 \begin{cases} x = 2 \\ x = -\frac{7}{2} \end{cases}$

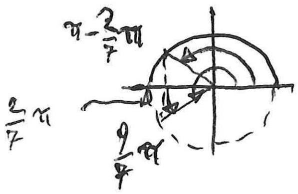
CHECK:

- $x = 2 \Rightarrow \log_5(2+3)$ AND $\log_5(2(2)-3)$ ARE DEFINED ✓
- $x = -\frac{7}{2} \Rightarrow \log_5\left(-\frac{7}{2}+3\right)$ UNDEFINED $\Rightarrow$ NOT A SOLUTION

ONLY ONE SOLUTION: $x = 2$.

3. Simplify the following expression, without approximating:
$\cancel{\sin^{-1}\left(\sin\left(\frac{8}{7}\pi\right)\right)}$ $\cos^{-1}\left(\cos\left(\frac{9}{7}\pi\right)\right)$

$\frac{9}{7}\pi > \pi$; $\cos\left(\frac{9}{7}\pi\right) = \cos\left(\pi - \frac{2}{7}\pi\right) = \cos\left(\frac{5}{7}\pi\right)$, WITH $0 \le \frac{5}{7}\pi \le \pi$



THEN
$\cos^{-1}\left(\cos\left(\frac{9}{7}\pi\right)\right) = \cos^{-1}\left(\cos\left(\frac{5}{7}\pi\right)\right) = \frac{5}{7}\pi$

(DO NOT APPROX TO 2.244)

4. Solve the following logarithmic equation. If needed, round your answer to 4 decimal places.

$\cancel{\log_5(x+3) + \log_5(2x-3) = 1}$
$\log_4(x-3) - \log_4(2x+3) = 2 \Rightarrow \log_4\left(\frac{x-3}{2x+3}\right) = 2 \overset{\text{EXPO}}{\Rightarrow} 4^{\text{LHS}} = 4^{\text{RHS}}$

$\Rightarrow \frac{x-3}{2x+3} = 4^2 \Rightarrow x-3 = 16(2x+3) \Rightarrow x-3 = 32x+48 \Rightarrow$
$\quad -x \; -48 \quad -x \; -48$

$\Rightarrow 31x = -51 \Rightarrow x = -\frac{51}{31}$

$\underline{\underline{\text{CHECK}}}$: $\log_4\left(-\frac{51}{31} - 3\right)$ DOES NOT EXIST $\Rightarrow x = -\frac{51}{31}$ EXTRANEOUS

SOLUT. $\Rightarrow$ <u>NONE SOLUTION</u>

MAT 121 – Exam3 – Spring 2019 – Take Home

Instructor: Dr. Francesco Strazzullo **Name**______________________________

I certify that I did not receive third party help in *completing* this test (sign) ______________________________

Instructions. Complete the following exercises. Each exercise is worth 10 points. If you need to approximate then **round to 3 decimal places**, unless otherwise specified. This is an open book test. You can also use a graphing tool and/or a computer algebra system like GeoGebra. When solving a problem graphically sketch the graph you used.
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5. Shawn just hopped on the edge of a merry-go-round. What are his linear and angular speeds if the diameter of the merry-go-round is 8 meters and it takes 10 seconds to make 3 complete revolutions? Round the solutions to two decimal places.

10 SECONDS FOR 3 LOOPS $\Rightarrow$ WHEN $T=10$, $\theta = 3(2\pi) = 6\pi \Rightarrow$

$\Rightarrow$ ANGULAR SPEED $= \omega = \frac{6\pi}{10} = \frac{3}{5}\pi \approx 1.88$ RAD/SEC.

LINEAR SPEED $= V = r\omega = \frac{8}{2} \cdot \frac{3}{5}\pi = \frac{12}{5}\pi \approx 7.54$ m/SEC.

6. Determine the amplitude, period, and phase shift of the following trigonometric equation.

$$y = 5\cos\left(-\frac{\pi}{6}x + \frac{2\pi}{7}\right) = a \cdot \cos(bx + c)$$

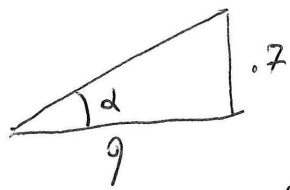
If there is no phase shift, state "no phase shift". If there is a phase shift, state the direction of the phase shift and the number of units (as a positive number) the graph is shifted.

AMPLITUDE $= |a| = 5$; PERIOD $= \frac{2\pi}{|b|} = \frac{2\pi}{\pi/6} = 12$;

PHASE-SHIFT $= \left|\frac{c}{b}\right| = \frac{2\pi/7}{\pi/6} = \frac{12}{7} \approx 1.714$

$\frac{c}{b} < 0 \Rightarrow$ SHIFT TO RIGHT.

7. Audrey is watching a space shuttle launch from an observation spot 9 miles away. Find the angle of elevation from her to the space shuttle, which is at a height of 0.7 miles. Write your answer in degrees rounded to two decimal places.



$$\tan\alpha = \frac{\text{OPP}}{\text{ADJ}} = \frac{.7}{9} \Rightarrow \alpha = \tan^{-1}\left(\frac{.7}{9}\right) \approx 4.45^\circ$$

(NOTE: IF YOU DO NOT CHANGE MODE FROM RAD: $\approx .08$ RAD)

8. Use trigonometric identities to simplify the expression and rewrite it in terms of one trigonometric function.
$\sec^2(\beta) + \csc^2(\beta)\,\cos^2(\beta)$

$$\| $$

$$\left(\frac{1}{\cos\beta}\right)^2 + \left(\frac{\cos(\beta)}{\sin(\beta)}\right)^2 = \frac{1}{\cos^2\beta} + \frac{\cos^2\beta}{\sin^2\beta} = \frac{1}{\cos^2\beta} + \frac{1-\sin^2\beta}{\sin^2\beta} =$$

$$= \frac{1}{\cos^2\beta} + \frac{1}{\sin^2\beta} - 1 = \frac{\sin^2 + \cos^2\beta}{\sin^2\beta\cdot\cos^2\beta} - 1 =$$

$$= \frac{1}{(\sin\beta\cdot\cos\beta)^2} - 1 = \frac{1}{\left(\frac{1}{2}\sin(2\beta)\right)^2} - 1 = \frac{4-\sin^2(2\beta)}{\sin^2(2\beta)}$$

NOTE: OTHER SIMPLIFICATIONS ARE POSSIBLE. $\sin^2(2\beta) = 1-\cos^2(2\beta) =$

$$= 1-(2\cos^2\beta-1)^2 = 1-4\cos^4\beta + 4\cos^2\beta - 1 = 4\cos^2\beta\,(1-\cos^2\beta).$$ THEN

- $4\csc^2(2\beta) - 1$ OR $\dfrac{1-\cos^2\beta(1-\cos^2\beta)}{\cos^2(1-\cos^2\beta)}$

9. (HONOR) In an effort to control vegetation overgrowth, 8 goats are released in an isolated area free of predators, but the limits of the area place a constraint on the population growth, whose limiting size is estimated to be 80 units. After 3 years, it is estimated that the goats' population has increased to 28. Assume logistic population growth, that is the population P after t years can be modeled by

$$P(t) = \frac{80}{1 + ae^{kt}}.$$

What will the population be at the beginning of the 29th year? (Use the greatest integer function rounding rule).

$$P(0) = 8 \Rightarrow \frac{80}{1+a} = 8 \Rightarrow 10 = 1 + a \Rightarrow a = 9$$

$$P(t) = \frac{80}{1+9e^{kt}}$$ THEN

$$28 = P(3) = \frac{80}{1+9e^{k(3)}} \Rightarrow 1 + 9e^{3k} = \frac{\cancel{80}^{20}}{\cancel{28}_{7}} \Rightarrow$$

$-1 \qquad -1$

$$\Rightarrow 9e^{3k} = \frac{13}{7} \Rightarrow e^{3k} = \frac{13}{63} \underset{\text{SIDES}}{\overset{\text{LN BOTH}}{\Rightarrow}} 3k = \ln\left(\frac{13}{63}\right)$$

$$\Rightarrow k = \frac{1}{3}\ln\left(\frac{13}{63}\right) \approx -.5261$$ THEN

$$P(t) = \frac{80}{1+9e^{\frac{1}{3}\ln\left(\frac{13}{63}\right)t}} = \frac{80}{1+9\left(\frac{13}{63}\right)^{\frac{1}{3}t}} \approx \frac{80}{1+9e^{-.5261t}}$$

$\lfloor x \rfloor$

$P(29) = 79$ GOATS

9. In an effort to control vegetation overgrowth, 8 goats are released in an isolated area free of predators. After 3 years, it is estimated that the goats' population has increased to 28. Assuming exponential population growth, what will the population be at the beginning of the 29th year? (Use the greatest integer function round rule).

EXPO GROWTH: $P = P_0 e^{kt}$

$P_0 = P(0) = 8$ GOATS. $\Big] \Rightarrow P = 8e^{kt}$

$28 = P(3) = 8e^{k(3)} = 8e^{3k} \Rightarrow e^{3k} = \frac{28}{8} = \frac{7}{2} \Rightarrow$

LN BOTH $\Rightarrow$ SIDES $\quad 3k = \ln\left(\frac{7}{2}\right) \Rightarrow k = \frac{1}{3}\ln\left(\frac{7}{2}\right) \approx .4176$ THEN

$P = 8e^{\frac{1}{3}\ln\left(\frac{7}{2}\right)t} = 8\left(\frac{7}{2}\right)^{\frac{1}{3}t} \approx 8e^{.4176t}$

$P(29) = 8e^{\frac{29}{3}\ln\left(\frac{7}{2}\right)} \approx 1{,}453{,}497$ GOATS $\approx 1{,}454{,}018$ GOATS