MAT 221 - Fall 2015 - Exam 2

Instructor: Dr. Francesco Strazzullo **Name** KEY

I did not receive third party help in completing this test. *Signature*____________

Instructions. You are expected to use a graphing calculator or software to complete some problems. Files can be downloaded and uploaded to the Eagleweb Coursework page for this assignment. Upload files or sketch any graph that you use or tables of input/output, **approximating up to the fourth decimal place**. Each problem is worth 10 points, unless otherwise specified. Available 100 points.
SHOW YOUR WORK NEATLY, PLEASE. (no work = no points)

1. Compute the first derivatives of the following functions: beside rules of differentiation, you might need techniques of differentiation like the logarithmic one. Show your work and box your answer in simplified expression. Each part is worth 10 points.

a. $u = v^{e^{3v}}$

LOG DIFFERENTIATION: $\ln u = \ln\left(v^{e^{3v}}\right) = e^{3v}\ln v$

$$\frac{d}{dv}[\ln u] = \frac{d}{dv}[e^{3v}\ln v] \Rightarrow \frac{u'}{u} = 3e^{3v}\cdot\ln v + e^{3v}\frac{1}{v} \Rightarrow$$

$$\Rightarrow u' = u\left(3\ln v + \frac{1}{v}\right)e^{3v} = \left(3\ln v + \frac{1}{v}\right)v^{e^{3v}}e^{3v}$$

b. $y = \log_5(x^2 - 3x)$

$$y' = \frac{d}{dx}[y] = \frac{1}{\ln 5}\cdot\frac{2x-3}{x^2-3x}$$

c. $s = \cos^{-1}(3t+1)$

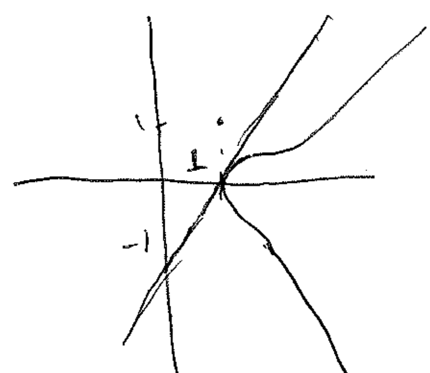
$$\frac{d}{dt}[\cos^{-1} u] = -\frac{u'}{\sqrt{1-u^2}} \rightarrow s' = -\frac{3}{\sqrt{1-(3t+1)^2}} = -\frac{3}{\sqrt{-9t^2-6t}}$$

d. $y = 2^{x-3x^2}$ $\rightarrow \frac{d}{dx}[a^u] = u' \cdot \ln a \cdot a^u$

$$y' = (1-6x)\ln 2 \cdot 2^{x-3x^2}$$

2. Find the equation of the line tangent to the curve with parametric equations $x = e^{2t} - t$, $y = t^3 + t$ at the point $(1, 0)$. Use symbolic notation.

NOTE THAT $(0,1)$ IS NOT ON THIS CURVE, THE SYSTEM $\begin{cases} e^{2t} - t = 0 \\ t^3 + t = 1 \end{cases}$ HAS NO SOLUTION. IF WE LOOK AT $(1,0)$, THE SYSTEM $\begin{cases} e^{2t} - t = 1 \\ t^3 + t = 0 \end{cases}$ IS SOLVED BY $t = 0$.



$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3t^2+1}{2e^{2t}-1}\Bigg|_{t=0} = \frac{1}{2-1} = 1$$

TANG. LINE:

$$L(x) = y_0 + y_0'(x - x_0) = 0 + 1(x-1)$$
$$= x - 1$$

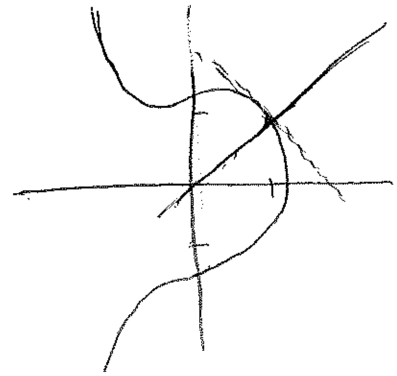
3. Write the equation of the normal line to the graph of $2x^3 - yx + 3y^2 = 4$ at the point $(1,1)$.

IMPLICIT DIFFERENTIATION

$$\frac{d}{dx}\left[2x^3 - yx + 3y^2\right] = \frac{d}{dx}[4]$$

$$6x^2 - (y'x + y\cdot 1) + 6y\cdot y' = 0$$

$$(6y - x)y' = y - 6x^2 \Rightarrow y' = \frac{y - 6x^2}{6y - x}$$



$$y'\big|_{(1,1)} = \frac{-5}{5} = -1$$

SLOPE OF NORMAL $= -\frac{1}{y'} = 1$

NORMAL LINE: $N(x) = y_0 - \frac{1}{y_0'}(x - x_0) = 1 + 1(x - 1) = x$

4. Consider the function $f(x) = x^2 - 2x - 1$. Compute $f'(2)$ in the following ways: (a) using the *definition by limit*; (b) by differentiation rules, and (c) with technology.

(a) $f'(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$

$f(2+h) - f(2) = (2+h)^2 - 2(2+h) - 1 - (2^2 - 2\cdot 2 - 1) =$

$= 4 + 4h + h^2 - 4 - 2h - 1 + 1 = h(2+h)$

$\to f'(2) = \lim_{h \to 0} \frac{\cancel{h}(2+h)}{\cancel{h}} \overset{\text{SUBS}}{=} 2 + 0 = 2$

(b) $f'(x) = 2x - 2 \to f'(2) = 2\cdot 2 - 2 = 2$

(c) WITH GGB: INPUT $f(x)$ AND TYPE $f'(2)$

WITH TI: $\boxed{Y=}$ $f(x)$ AND TYPE $\boxed{2^{ND}}$, $\boxed{\text{TRACE}}$, $\boxed{6}$.

5. Compute the following limit

$$\lim_{x \to -2} \frac{\sqrt{21+x^2}-5}{x+2} \stackrel{\text{SUBS}}{=} \frac{0}{0} \text{ UNDEFINE} \to \text{SIMPLIFY}$$

$$\frac{\sqrt{21+x^2}-5}{x+2} \cdot \frac{\sqrt{21+x^2}+5}{\sqrt{21+x^2}+5} = \frac{21+x^2-25}{(x+2)(\sqrt{21+x^2}+5)} = \frac{(x-2)(x+2)}{\cancel{(x+2)}(\sqrt{21+x^2}+5)}$$

$$\lim_{x \to -2} \frac{\sqrt{21+x^2}-5}{x+2} = \lim_{x \to -2} \frac{x-2}{\sqrt{21+x^2}+5} \stackrel{\text{SUBS}}{=} \frac{-2-2}{\sqrt{21+4}+5} = -\frac{4}{10}$$

$$= -\frac{2}{5}$$

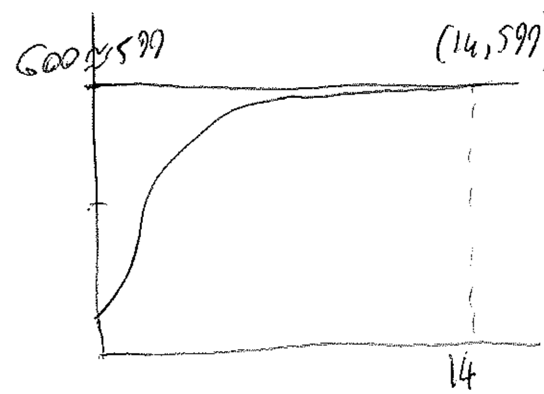
6. A population of 15 bacteria is introduced into a culture. The number of bacteria is modeled by

$$P(t) = \frac{1800t^2 + 15}{3t^2+1}$$

where t is measured in hours. What is the limiting size L (or *carrying capacity*) of this population? Approximately (to the nearest integer), after how many hours can you say that the population has reached its carrying capacity, that is $P = L - 1$?

$$L = \lim_{t \to \infty} P(t) = \lim_{t \to \infty} \frac{1800t^2+15}{3t^2+1} \underset{\text{SAME DEGREE}}{\overset{\text{POLY. WITH}}{=}} \frac{1800}{3} = 600$$

THEN THE POPULATION LIMITING SIZE IS 600 BACTERIA



USING TECHNOLOGY WE SOLVE THE EQUATION $P(t) = 599$, WHICH GIVES $t = 13.95 \approx 14$.

NOTE THAT SOLVING $P(t) = 598.5$ IS NOT CORRECT, BECAUSE IT WOULD BE CONSIDERING ONLY 598 BACTERIA.

7. The following table includes the number of earned BS degrees in USA from 2000 to 2011, with specifics about Computer Sciences and Mathematics (check the files on EagleWeb).

Earned bachelor's degree, 2000–11

	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
All fields	1,254,618	1,260,308	1,308,970	1,365,694	1,417,421	1,456,401	1,502,922	1,541,704	1,580,413	1,619,028	1,668,227	1,734,229
Computer sciences	37,519	43,597	49,706	57,926	59,968	54,588	48,000	42,596	38,922	38,496	40,107	43,586
Mathematics	11,714	11,437	12,254	12,863	13,735	14,816	15,310	15,551	15,841	16,208	16,832	18,021
Ratios CS	2.99%	3.46%	3.80%	4.24%	4.23%	3.75%	3.19%	2.76%	2.46%	2.38%	2.40%	2.51%
Ratios Math	0.93%	0.91%	0.94%	0.94%	0.97%	1.02%	1.02%	1.01%	1.00%	1.00%	1.01%	1.04%

Source: http://www.nsf.gov/statistics/seind14/content/chapter-2/at02-23.xls

a. Use the above table to approximate the rate of change of the percent of earned Computer Science degrees in 2008 (include units). USE X AS IN PART (b)

$$f'(8) \approx \frac{1}{2}\left(\frac{f(8)-f(7)}{8-7} + \frac{f(9)-f(8)}{9-8}\right) = \frac{1}{2}(-.3 - .08) = -.19 \text{ \% PER YEAR}$$

b. Compute the 5[th] degree polynomial regression and the trigonometric (sine) regression that best fit the percent of earned Computer Science degrees as a function of the year, **considering years from 2000**. Report the correlation coefficients (round to 4 decimal places) and state which regression line is the better model in this context. $X = \text{YEAR} - 2000: 0, 1, 2, \dots, 11$

POLYNOMIAL: $y = -.0005x^5 + .0141x^4 - .1286x^3 + .3503x^2 + .1656x + 2.9997$

SINE: $y = 3.1887 + .9491\sin(.5345x - .2129)$, $R^2 = .9787$

BEST MODEL (ACCORDING TO R^2) IS THE POLYNOMIAL. $R^2 = .9931$

c. Use the best model from part b. to estimate the rate of change of the percent of earned Computer Science degrees in 2008, then compare these to your results from part a.

$f'(8) = -.21$ PERCENT PER YEAR.

THE APPROXIMATION IN (a) IS ALMOST ACCURATE AND THE TWO VALUES BOTH INDICATE A DECREASING TREND.