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Name VSY

Instructions. Only calculators are allowed on this examination. Each problem is worth 10 points. Always use the appropriate wording and units of measure in your answers (when applicable). **SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).**

1. Perform the indicated operation on the rational expressions and reduce your answer to lowest terms.

$$\begin{aligned} & \frac{36(2x+3)(3x-1)}{7(9x^2-1)} \div \frac{28(2x-3)(3x+1)}{9(4x^2-9)} = \\ & = \frac{36(2x+3)(3x-1)}{7(3x+1)(3x-1)} \cdot \frac{9(2x-3)(2x+3)}{28(2x-3)(3x+1)} = \frac{9 \cdot 4 \cdot 9}{7 \cdot 4 \cdot 7} \frac{(2x+3)^2}{(3x+1)^2} \\ & = \frac{81}{49} \frac{(2x+3)^2}{(3x+1)^2} = \frac{81(4x^2+12x+9)}{49(9x^2+6x+1)} \end{aligned}$$

2. Perform the indicated operation on the rational expressions and reduce your answer to lowest terms.

$$\begin{aligned} & \frac{z-5}{z^2-2z-15} \cdot \frac{z^2+4z-21}{z^2-6z+9} = \\ & = \frac{\cancel{z-5}}{(z-5)(z+3)} \cdot \frac{(z-3)(z+7)}{(z-3)^2} = \frac{z+7}{(z+3)(z-3)} \\ & = \frac{z+7}{z^2-9} \end{aligned}$$

3. Perform the indicated operation on the two rational expressions and reduce your answer to lowest terms.

$$\begin{aligned} & \frac{4x^2}{x^2-4} + \frac{3}{x+2} = \frac{4x^2}{(x-2)(x+2)} + \frac{3}{x+2} = \\ & = \frac{4x^2+3(x-2)}{(x-2)(x+2)} = \frac{4x^2+3x-6}{(x-2)(x+2)} \end{aligned}$$

$$\text{LCD} = (x-2)(x+2)$$

$$\begin{aligned} \text{Product} &= (-6) \cdot (4) = -24 \rightarrow \pm(24, -1); \pm(12, -2); \\ \text{Sum} &= 3 \quad \pm(8, -3); \pm(6, -4); \\ &\quad \text{NONE SUMS TO 3} \end{aligned}$$

4. Consider the following equation:

$$\frac{9x^2}{(x-3)(x+2)} + \frac{3}{x-3} = 9$$

Step 1. State any restriction(s) on the variable. If a restriction is not needed, write "No Restriction".

$$x-3=0 \rightarrow x=3$$

$$x+2=0 \rightarrow x=-2$$

Step 2. Solve the equation and simplify your answer. Write your solution as either an integer or a simplified fraction. If the equation has no solution, write "No Solution".

L.C.D. = $(x-3)(x+2)$ MULTIPLY THE EQUATION

$$9x^2 + 3(x+2) = 9(x-3)(x+2) \rightarrow 9x^2 + 3x + 6 = 9(x^2 - 3x + 2x - 6) \rightarrow 9x^2 + 3x + 6 = 9x^2 - 9x - 54 \rightarrow 12x + 60 = 0$$

$$\rightarrow x = -\frac{60}{12} = -5 \text{ NOT A RESTRICTED VALUE.}$$

$$\text{CHECK: } \frac{9 \cdot (-5)^2}{(-5-3)(-5+2)} + \frac{3}{-5-3} = \frac{9 \cdot 25}{8 \cdot -8} - \frac{3}{8} = \frac{3}{8}(25-1) = 9 \checkmark$$

5. The volume of a gas in a container varies inversely with the pressure on the gas. If a gas has a volume of 240 cubic inches under a pressure of 7 pounds per square inch, what will be its volume if the pressure is increased to 10 pounds per square inch? (Round off your answer to the nearest cubic inch.)

INVERSELY PROPORTIONAL: $P \cdot V = K$ CONSTANT
 GIVEN CONDITION: $P=7, V=240$ $\rightarrow K = 7 \cdot 240 = 1680$

EQUATION: $PV = 1680$
 QUESTION: $P=10, V=?$ $\rightarrow 10 \cdot V = 1680 \rightarrow V = \frac{1680}{10}$

$V = 168$ CUBIC INCHES WHEN $P = 10$ lb.

ALTERNATIVE VERSION

6. Jimmy and John, working together, can paint a 300 ft long standard fence in 6 hours. Working alone, Jimmy takes 3 hours and half more than John to complete the same job. How long does it take John to paint the same fence alone?

JOBS	TIME HOURS	WORK RATES
Jimmy	$X+3.5$	$\frac{1}{X+3.5}$
John	X	$\frac{1}{X}$
TOTAL	6	$\frac{1}{6}$

$$\frac{1}{X+3.5} + \frac{1}{X} = \frac{1}{6}$$

MULTIPLY BY LCD: $6(X+3.5) \cdot X$

$$6X + 6(X+3.5) = (X+3.5)X$$

$$6X + 6X + 21 = X^2 + 3.5X$$

$$X^2 + 3.5X - 12X - 21 = 0$$

$$X^2 - \frac{17}{2}X - 21 = 0 \quad \text{MULTIPLY BY 2}$$

$$2X^2 - 17X - 42 = 0$$

$$\text{PRODUCT} = 2 \cdot (-42)$$

$$2X^2 - 21X + 4X - 42 = 0$$

$$= -84$$

$$\text{SUM} = -17$$

$$X(2X-21) + 2(2X-21) = 0 \quad (-21, 4) \Delta$$

$$(X+2)(2X-21) = 0$$

$$X = -2$$

NOT
IN CONTEXT

$$X = \frac{21}{2} = 10.5 \text{ HOURS}$$

7. Simplify the expression by combining the radical terms using the indicated operation(s). Assume all variables are positive.

$$6a\sqrt{100ab} - 5a\sqrt{4ab} + 5a\sqrt{144ab}$$

6. Jimmy and John, working together, can paint a 300 ft long standard fence in 4 hours. Working alone, Jimmy takes 1 hour and half more than John. How long does it take John to paint the same fence alone?

JOB	TIME HOUR	WORK-RATE
JIMMY	$X+1.5$	$\frac{1}{X+1.5}$
JOHN	X	$\frac{1}{X}$
TOTAL	4	$\frac{1}{4}$

NOTE: "ONE HOUR AND HALF MORE THAN"

$$\frac{1}{X+1.5} + \frac{1}{X} = \frac{1}{4}$$

$$\text{LCD} = 4X(X+1.5)$$

$$4X + 4(X+1.5) = X(X+1.5)$$

$$4X + 4X + 6 = X^2 + 1.5X$$

$$X^2 + \frac{3}{2}X - 8X - 6 = 0 \quad \text{MULTIPLY BY 2}$$

$$2X^2 + 3X - 16X - 12 = 0$$

$$\rightarrow 2X^2 - 13X - 12 = 0$$

WE CAN'T FACTOR.

WE CAN'T FIND A SOLUTION

TO THIS PROBLEM.

$$\text{PRODUCT} = 2(-12) = -24$$

$$\text{SUM} = -13$$

$$\pm(-24, 1); \pm(-12, 2)$$

$$\pm(-8, 3); \pm(-6, 4)$$

NONE ADDS TO -13

7. Simplify the expression by combining the radical terms using the indicated operation(s). Assume all variables are positive.

$$6a\sqrt{100ab} - 5a\sqrt{4ab} + 5a\sqrt{144ab}$$

$$a(6\sqrt{100}\sqrt{ab} - 5\sqrt{4}\sqrt{ab} + 5\sqrt{144}\sqrt{ab})$$

$$a(6 \cdot 10 - 5 \cdot 2 + 5 \cdot 12)\sqrt{ab}$$

$$110a\sqrt{ab}$$

8. Simplify the following radical by rationalizing the denominator.

$$\frac{(\sqrt{y}+2)}{(\sqrt{y}-2)} \cdot \frac{(\sqrt{y}+2)}{(\sqrt{y}+2)} = \frac{(\sqrt{y}+2)^2}{(\sqrt{y})^2 - 2^2} = \frac{(\sqrt{y})^2 + 2 \cdot 2\sqrt{y} + 2^2}{y - 4}$$

$$= \frac{y + 4 + 4\sqrt{y}}{y - 4}$$

9. Use the Pythagorean Theorem to determine (and show) whether or not the triangle with the following sides is a right triangle.

a: 23, b: 27, c: 35

A) Yes

~~B) No~~

$$a^2 + b^2 \stackrel{?}{=} c^2 = 35^2 = 1225$$

$$23^2 + 27^2 = 1258 \quad \neq$$

10. Find the distance between the points given.

$(-5, 2), (3, -6)$

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$= \sqrt{(-5 - 3)^2 + (2 - (-6))^2}$$

$$= \sqrt{8^2 + 8^2} = \sqrt{2 \cdot 8^2} = \sqrt{8^2} \cdot \sqrt{2}$$

$$= 8\sqrt{2} \approx 11.31$$