Math 102 - Fall 2011 - Final

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Instructions. Only calculators are allowed on this examination. **Each problem is worth 10 points, unless otherwise specified.**

Always use the appropriate wording and units of measure in your answers (when applicable). You might need the following formulas:

$$S = P\left(1+\frac{r}{k}\right)^{kt}, \quad S = P(1+i)^n, \quad S = Pe^{rt}, \quad S = \frac{R}{i}\left((1+i)^n - 1\right), \quad A = \frac{R}{i}\left(1-(1+i)^{-n}\right).$$

SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

1. (15 points) The following table gives the percentage of students with passing grade in MAT 102 for selected academic years.

Year	2000	2002	2004	2006	2007	2008	2009	2010
% passing	75	76	79	80	84	85	89	90

(a) Using your calculator with x equal to the number of years past 2000 and y the percent of passing students, find the quartic model and the exponential model which best fit these data and use the *correlation coefficient* in order to decide which model is better. Report your models and coefficients approximated to the third decimal place.

EXPO: $Y = 73.559(1.019^X)$, $R^2 = .94227$

QUART.: $Y = -.004X^4 + .0842X^3 - .398X^2 + 1.296X + 74.896$, $R^2 = .98$

QUARTIC IS THE BEST.

(b) Use the unrounded best model from the previous part in order to interpolate and to extrapolate the percentage of students passing MAT 102 in two years of your choice.

INTERPOLATION: $2005 \to X = 5$, $Y = 79$

EXTRAPOLATION: $2011 \to X = 11$, $Y = 92$ (OR 91.6)

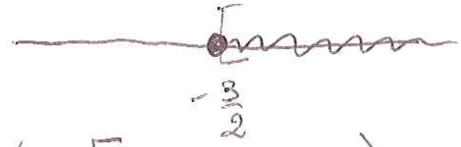
2. Give the solution in interval notation for the inequality $x - 1 \leq 3x + 2$.

$X - 1 \leq 3X + 2$

$-3X + 1 \quad -3X + 1$

$-2X \leq 3$

$\frac{-2X}{-2} \geq \frac{3}{-2} \to X \geq -\frac{3}{2}$



SOLUTION $\left[-\frac{3}{2}, +\infty\right)$

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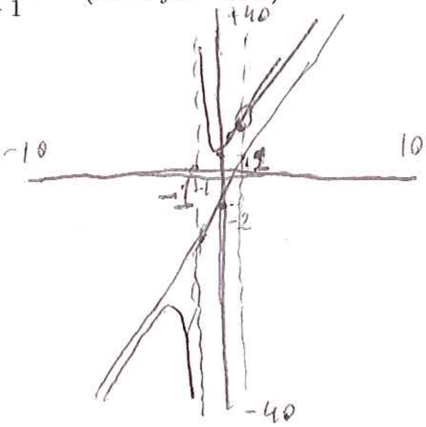
3. Find the domain and all (if any) asymptotes of the function $y = \dfrac{3x^3 - 2x^2 - 1}{x^2 - 1}$. (Show your work)



DOMAIN: $x^2 - 1 \neq 0 \rightarrow x^2 \neq 1 \rightarrow x \neq \pm 1$

$\rightarrow (-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

CHECK V.A: I) $x = 1 \xrightarrow{\text{PLUG}} \frac{0}{0} \rightarrow$ NOT V.A.

II) $x = -1 \rightarrow \dfrac{3(-1)^3 - 2(-1)^2 - 1}{0} = \dfrac{-6}{0} \rightarrow$ V.A.

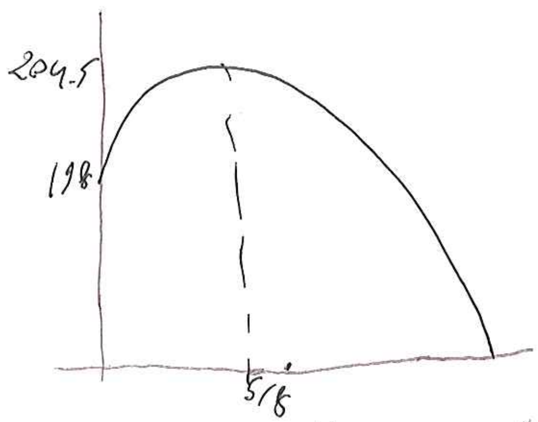
HORIZ./SLANT:

$$\begin{array}{r|l} & 3x - 2 \\ x^2 - 1 & 3x^3 - 2x^2 - 1 \\ & 3x^3 - 3x \\ \hline & -2x^2 + 3x - 1 \\ & -2x^2 + 2 \\ \hline & 3x - 3 \end{array}$$

$y = 3x - 2 + \dfrac{3x - 3}{x^2 - 1}$

$\rightarrow$ SLANT ASIMP.: $y = 3x - 2$

4. The vertical displacement of a free falling rock can be described by the function $s(x) = -16t^2 + 20t + 198$ (measured in feet), where t is the time in seconds after the rock has been thrown. What is the maximum altitude that this rock will reach? How long does it take the rock to reach its highest altitude?



PLUG IT IN CALCULATOR, WITH A WINDOW $[0, 250] \times [0, 5]$

$\boxed{\text{2ND}} + \boxed{\text{TRACE}} + \boxed{\text{MAXIMUM}}$ GIVES

$x = .625$, $y = 204.25$

ALGEBRAIC: "MAXIMUM" $\equiv$ "VERTEX" (h, k)

$h = \dfrac{-b}{2a} = \dfrac{-20}{2(-16)} = \dfrac{5}{8}$ SECONDS

$k = s(h) = s(5/8) = 204.25$ FEET

10 (3)

5. Use a calculator to find the following numbers.

(a) $\log_6 42 =$ 2.086

(b) $\log_{0.1} 1.4 =$ $-.14613$

(c) $10^{2/5} =$ 2.51189

6. A couple wants to establish a fund that will provide \$6000 for college expenses at the end of each 3-month period for 5 years. If a lump sum can be placed in an account that pays 6.5% compounded quarterly, what lump sum is required?

THIS IS AN ANNUITY (RECURRING PAYMENTS) WITH: $R = 6000$, $t = 5$, $K =$ "QUARTERLY" $= 4$, $r = .065 \rightarrow i = \frac{r}{K} = .01625$, $n = r \cdot K = 20$.

BECAUSE THE LUMP SUM HAS TO BE PLACED NOW, THEN WE COMPUTE THE PRESENT VALUE:

$$A = \frac{R}{i}\left(1-(1+i)^{-n}\right) = \frac{6000}{.01625}\left(1-(1+.01625)^{-20}\right)$$

$$= \$\ 101\,753.60$$

7. Mr Carter invests \$300 at the end of each month in an account that pays 5% compounded monthly. How much will be in the account in 25 years?

ANNUITY (RECURRING PAYMENTS) TO "CASH OUT" AFTER 25 YEARS (FUTURE VALUE)

$R = 300$, $r = .05$, $K =$ "MONTHLY" $= 12$, $t = 25 \rightarrow i = \frac{.05}{12}$, $n = 12 \cdot 25 = 300$

$$S = \frac{R}{i}\left((1+i)^n - 1\right) = \frac{300}{.05/12}\left(\left(1+\frac{.05}{12}\right)^{300} - 1\right)$$

$$= \$\ 178\,652.91$$

30

8. (15 points) You figure out a new diet, taking in account three food kinds, dairy, meat, and vegetables. Dairy products considered contain 4 calories per gram. Vegetables considered contain 1 calory per gram. Meat considered contain 8 calories per gram. You want a daily intake of 2000 calories, with exactly 500 grams of daily food given by dairy and vegetables. Moreover, you always want the serving size of vegetable to be the same as the sum of the serving sizes of meat and dairy. Let d, m, and v, measured in grams, be the serving sizes of respectively dairy, meat, and vegetables. Write a system of equations modeling your diet and then find the serving sizes. (If using a calculator, write the augmented matrix of the system and the corresponding reduced row echelon form.)

ITEMS	QUANTITIES IN GRAMS	RATES IN CAL/g	QR CALORIES
DAIRY	d	4	4d
MEAT	m	8	8m
VEGGY	v	1	v
TOTALS	500		2000

$V = d + m \to$

$\to d + m - V = 0$

3X4 MATRIX

$$\begin{cases} d + m + V = 500 \\ 4d + 8m + V = 2000 \\ d + m - V = 0 \end{cases} \to \left[\begin{array}{ccc|c} 1 & 1 & 1 & 500 \\ 4 & 8 & 1 & 2000 \\ 1 & 1 & -1 & 0 \end{array}\right] \xrightarrow{\text{RREF}}$$

$$\to \left[\begin{array}{ccc|c} 1 & 0 & 0 & 62.5 \\ 0 & 1 & 0 & 187.5 \\ 0 & 0 & 1 & 250 \end{array}\right] \to \begin{cases} d = 62.5 \\ m = 187.5 \\ V = 250 \end{cases}$$

GRAMS

9. Solve the following systems of linear equations. If using a calculator, write the augmented matrix of the system and the corresponding reduced row echelon form.

$$\begin{cases} x - 3y + 2z = 12 \\ 2x - 6y + z = 7 \end{cases} \xrightarrow[2\times 3]{} \left[\begin{array}{ccc|c} 1 & -3 & 2 & 12 \\ 2 & -6 & 1 & 7 \end{array}\right] \xrightarrow{\text{RREF}}$$

$$\to \left[\begin{array}{ccc|c} 1 & -3 & 0 & 2/3 \\ 0 & 0 & 1 & 17/3 \end{array}\right] \to \begin{cases} X - 3Y = 2/3 \to X = 3Y + \frac{2}{3} \\ Z = 17/3 \end{cases}$$

$$\text{SOLUTION:} \begin{cases} X = 3Y + 2/3 \\ Y = Y \\ Z = 17/3 \end{cases}$$

25