Math 102 - Fall 2011 - Test 4

Instructor: Dr. Francesco Strazzullo Name KEY

Instructions. Only calculators are allowed on this examination. **Each problem is worth 15 points, unless otherwise specified. There are 80 points available and your score will be a rounded-up percentage of these points.**

Always use the appropriate wording and units of measure in your answers (when applicable). You might need the following formulas:

$$S = P\left(1+\frac{r}{k}\right)^{kt}, \quad S = P(1+i)^n, \quad S = Pe^{rt}, \quad S = \frac{R}{i}\left[(1+i)^n - 1\right], \quad A = \frac{R}{i}\left[1-(1+i)^{-n}\right].$$

SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

1. Solve the following logarithmic equation and check for *extraneous solutions*

$2\log_3(3-x) - \log_3(7-2x) = 1$

$\log_3(3-x)^2 - \log_3(7-2x) = 1 \rightarrow \log_3\left(\frac{(3-x)^2}{7-2x}\right) = 1$

$\rightarrow 3^{\log_3 \frac{(3-x)^2}{7-2x}} = 3^1 \rightarrow \frac{(3-x)^2}{7-2x} = 3 \rightarrow$

$\rightarrow (3-x)^2 = 3(7-2x) \rightarrow 3^2 - 2(3x) + x^2 = 21 - 6x$

$-9 \quad +6x \qquad\qquad -9 \quad +6x$

$\rightarrow x^2 = 12 \rightarrow x = \pm\sqrt{12} = \pm 2\sqrt{3} \approx \pm 3.46$

CHECK: 1) $x = 2\sqrt{3} \rightarrow 3-(2\sqrt{3}) < 0 \rightarrow 2\log_3(3-x)$ IS NOT DEFINED

$\rightarrow x = 2\sqrt{3}$ IS EXTRANEOUS.

2) $x = -2\sqrt{3} \rightarrow 3-x$ AND $7-2x$ ARE BOTH POSITIVE AND LOGS ARE DEFINED.

ONLY ONE POSSIBLE SOLUTION: $x = -2\sqrt{3}$

2. (10 points) Use a calculator to find the value of the following logarithms.

(a) $\log_4 8 =$ $\frac{\ln 8}{\ln 4} = 1.5 = \frac{3}{2}$

(b) $\log_{.75} 0.5 =$ $\frac{\ln 0.5}{\ln 0.75} \approx 2.4094$

3. (10 points) Without using a calculator, find the value of the following logarithms.

(a) $\log_2 256 =$ $\log_2 2^8 = 8$

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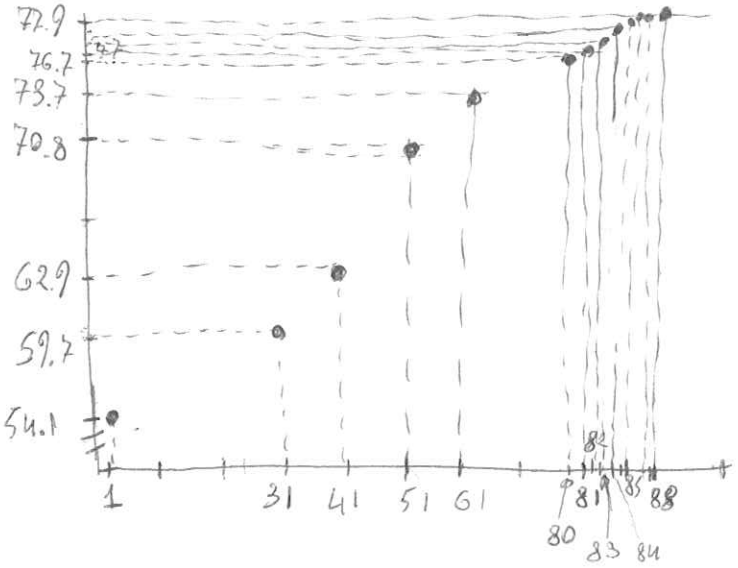
(b) $\log_4 \frac{1}{16} =$ $\log_4 \frac{1}{4^2} = \log_4 4^{-2} = -2$

4. The table below gives the life expectancy for people in the United States for selected birth years from 1920 to 2007.

X	Year	Life Span (years)	X	Year	Life Span (years)	X	Year	Life Span (years)
1	1920	54.1	61	1980	73.7	83	2002	77.3
31	1930	59.7	80	1999	76.7	84	2003	77.5
41	1940	62.9	81	2000	77	85	2004	77.8
51	1970	70.8	82	2001	77.2	88	2007	77.9

(a) Consider x to be the birth year, counting from 1920, and y to be the expected life span, then draw a scatter-plot of the given data.

IN PART (b) IT IS ASKED FOR A LN MODEL. AND WE CAN'T USE $X=0$; $\therefore$ 1920 $\rightarrow X=1$



(b) Using your calculator, find both the quadratic and the logarithmic models which best fit the data. Report your answer to the third decimal place. Report and use the *correlation coefficients* to say which of these models is the best fit for the given data.

QUAD: $Y = -.0002X^2 + .315X + 52.703$, $R^2 = .9638$

LOG: $Y = 50.41 + 5.584 \ln X$, $R^2 = .7137$

THE BEST ONE IS THE QUADRATIC

(c) Use the unrounded best model to compute examples of interpolation and extrapolation from the given data.

INTERPOLATION $\rightarrow$ INSIDE DATA $\rightarrow$ 1985 $\rightarrow X=66 \rightarrow Y=72.6$

EXTRAPOLATION $\rightarrow$ OUTSIDE DATA $\rightarrow$ 2011 $\rightarrow X=92 \rightarrow Y=80$

5. Your new employer offers you two options for an early contract-termination benefit:

   (a) a one time payment of \$43000, or

   (b) a monthly payment of \$1800 for two years.

   If you can assume a constant inflation rate of 3.5%, which option is more valuable for you when (and if) your contract is terminated early? Why?

WE MUST COMPARE (a) WITH THE PRESENT VALUE OF (b):

(b) IS AN ANNUITY WITH $R = 1800$, $K = 12$, $t = 2$ AND $Y = 3.5\% = .035$

$$A = \frac{R}{Y/K}\left(1-\left(1+\frac{Y}{K}\right)^{-Kt}\right) = \frac{1800}{.035/12}\left(1-\left(1+\frac{.035}{12}\right)^{-12\cdot 2}\right)$$

$$= \$41664.04 \quad (\text{NOTE ACTUAL PAYMENT} = R\cdot n = 1800\cdot 24 = \$43200)$$

ANSWER: OPTION (a) IS MORE VALUABLE (WORTH \$43000) THAN OPTION (b) (WORTH \$41664.04) AT CONTRACT TERMINATION.

6. You are shopping for a simple investment of \$3000 that you expect to cash in four years. They offer you two options:

   (a) a daily compounded account at 4.5%;

   (b) a continuously compounded account at 4.25%.

   Compute which option is the best deal for you.

IN BOTH CASES WE HAVE $P = 3000$ AND $t = 4$.

(a) $$S = P\left(1+\frac{Y}{K}\right)^{Kt} = 3000\left(1+\frac{.045}{365}\right)^{365\cdot 4} = \$3591.61$$

$K = \text{"DAILY"} = 365$, $Y = 4.5\% = .045$

(b) $$S = Pe^{Yt} = 3000\, e^{.0425(4)} = \$3555.92$$

$Y = 4.25\% = .0425$

ANSWER: OPTION (a) IS THE BEST DEAL.