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Name Key

Instructions. Only calculators are allowed on this examination. Each problem is worth 10 points. Always use the appropriate wording and units of measure in your answers (when applicable). **SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).**

1. Perform the indicated operation of multiplication or division on the rational expressions and simplify.

$$\frac{25a^2}{2b^3} \cdot \frac{4b^2}{5a^2}$$

$$\frac{5^2 \cdot 2^2}{2 \cdot 5} a^{2-2} b^{2-3} = 5 \cdot 2 \cdot b^{-1} = \boxed{\frac{10}{b}}$$

2. Perform the indicated operation of multiplication or division on the rational expressions and simplify.

$$\frac{y+9}{y^2+4y-21} \cdot \frac{y^2-7y+12}{y+9}$$

$$\frac{\cancel{y+9}}{(y+7)(\cancel{y-3})} \cdot \frac{(y-4)(\cancel{y-3})}{\cancel{y+9}} = \boxed{\frac{y-4}{y+7}}$$

Product = -21
Sum = 4

Product = 12
Sum = -7

3. Perform the indicated operation of addition on the two rational expressions and reduce your answer to lowest terms.

$$\frac{4x^2}{x^2-4} + \frac{3}{x+2}$$

$$\frac{4x^2}{(x-2)(x+2)} + \frac{3}{x+2} = \frac{4x^2 + 3(x-2)}{(x-2)(x+2)} = \boxed{\frac{4x^2 + 3x - 6}{(x-2)(x+2)}}$$

LED = (x-2)(x+2)

Product = (-6) · 4 = -24
Sum = 3

-1, 24 ; -2, 12 ; -3, 8
-4, 6 NOT FACTORABLE

4. Consider the following equation:

$$\frac{-2}{x-2} + \frac{3}{x+8} = 1$$

Step 1. State any restriction(s) on the variable. If a restriction is not needed, write "No Restriction".

$$x-2 \neq 0 \text{ AND } x+8 \neq 0 \rightarrow x \neq 2 \text{ AND } x \neq -8$$

Step 2. Solve the equation and simplify your answer. Write your solution as either an integer or a simplified fraction. If the equation has no solution, write "No Solution".

$$\text{LCD} = (x-2)(x+8) \rightarrow -2(x+8) + 3(x-2) = 1(x-2)(x+8)$$

$$-2x-16+3x-6 = x^2+6x-16 \rightarrow x-22 = x^2+6x-16 \rightarrow$$

$$\rightarrow x^2+5x+6=0 \rightarrow (x+3)(x+2)=0 \quad \begin{matrix} x+3=0 \rightarrow x=-3 \\ \text{OR} \\ x+2=0 \rightarrow x=-2 \end{matrix}$$

$$\text{CHECK: } x=-2: \frac{-2}{(-2)-2} + \frac{3}{(-2)+8} = \frac{-2}{-4} + \frac{3}{6} = \frac{1}{2} + \frac{1}{2} = 1 \checkmark$$

$$x=-3: \frac{-2}{(-3)-2} + \frac{3}{(-3)+8} = \frac{-2}{-5} + \frac{3}{5} = \frac{2}{5} + \frac{3}{5} = \frac{5}{5} = 1 \checkmark$$

5. An inlet pipe on a swimming pool can be used to fill the pool in 5 hours. The drain pipe can be used to empty the pool in 6 hours. If the pool is $\frac{2}{5}$ filled and then the inlet pipe and drain pipe are opened, how long from that time will it take to fill the pool?

JOB:	TIME	WORK RATE
FILL POOL	HOURS	1/HOUR
INLET	5	$\frac{1}{5}$
DRAIN (AGAINST)	6	$-\frac{1}{6}$
TOTAL (FILL EMPTY POOL)	X	$\frac{1}{X}$

$$\frac{1}{5} - \frac{1}{6} = \frac{1}{X} \rightarrow \frac{1}{30} = \frac{1}{X} \rightarrow X=30$$

ANSWER = "TIME FOR FILLING A 2 PORTION OF POOL" = $2 \cdot X$

$$\text{"WHOLE POOL": } 1 = 2 + \frac{2}{5} \rightarrow 2 = 1 - \frac{2}{5} = \frac{3}{5}$$

$$\rightarrow \text{ANSWER} = \frac{3}{5} (30) = \boxed{18 \text{ HOURS}}$$

6. Pam and Craig, working together, can mow the lawn in 6 hours. Working alone, Craig takes twice as long as Pam. How long does it take Pam to mow the lawn alone?

WORKING	TIME	WORK RATE
PAM	x	$1/x$
CRAG	$2x$	$1/2x$
TOGETHER (TOTAL)	6	$1/6$

$$\frac{1}{x} + \frac{1}{2x} = \frac{1}{6}$$

LCD: $6 \cdot x$

$$\rightarrow 6 + 3 = x \rightarrow \boxed{x = 9 \text{ HOURS}}$$

7. Simplify the expression by combining the radical terms using the indicated operation(s). Assume all variables are positive.

$$7x\sqrt{54xy} - x\sqrt{6xy} - 8x\sqrt{24xy}$$

$$7x\sqrt{9}\sqrt{6xy} - x\sqrt{6xy} - 8x\sqrt{4}\sqrt{6xy}$$

$$(7x(3) - x - 8x(2))\sqrt{6xy}$$

$$(21x - x - 16x)\sqrt{6xy}$$

$$\boxed{4x\sqrt{6xy}}$$

8. Simplify the following radical by rationalizing the denominator.

$$\frac{\sqrt{y}}{\sqrt{y}+5} \cdot \frac{\sqrt{y}-5}{\sqrt{y}-5} = \frac{\overset{\sqrt{y}}{\sqrt{y}+5} (\sqrt{y})^2 - 5\sqrt{y}}{(\sqrt{y})^2 - 5^2} = \boxed{\frac{y - 5\sqrt{y}}{y - 25}}$$

9. Use the Pythagorean Theorem to determine whether or not the triangle with the following sides is a right triangle.

a: 48, b: 64, c: 80

~~A) Yes~~

B) No

$$\begin{aligned} (48)^2 + (64)^2 &= 2304 + 4096 \\ &= 6400 = 80^2 \quad \checkmark \end{aligned}$$

10. Find the distance between the points given.

$(-4, 3), (2, -1)$

$$\begin{aligned} d &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\ &= \sqrt{(-4 - 2)^2 + (3 - (-1))^2} \\ &= \sqrt{6^2 + 4^2} \\ &= \sqrt{52} = \sqrt{4} \cdot \sqrt{13} \\ &= 2\sqrt{13} \approx 7.21 \end{aligned}$$