Math 102 - Spring 2010 - Test 2

Instructor: Dr. Francesco Strazzullo Name KEY

Instructions. Only calculators are allowed on this examination. Each problem is 10 points worth. **Always use the appropriate wording and units of measure in your answers (when applicable). SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).**

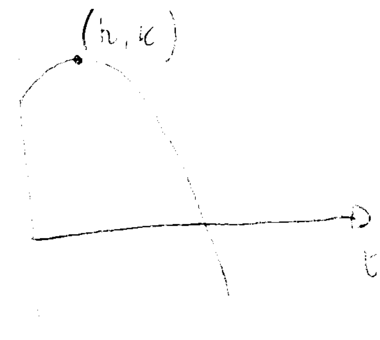
1. Find the x-coordinate of the vertex of the parabola $y = .35x^2 + 70x - 1$.

$$x = -\frac{b}{2a} = -\frac{70}{2 \cdot .35} = -100$$

2. The vertical displacement of a free falling rock can be described by the function $s(x) = -16t^2 + 20t + 198$ (measured in feet), where t is the time in seconds after the rock has been thrown. What is the maximum altitude that this rock will reach? How long does it take the rock to reach its highest altitude?

$$s(x) = -16t^2 + 20t + 198$$

DOWNWARDS PARABOLA



MAXIMUM HIGHT AT VERTEX

$$h = -\frac{b}{2a} = \frac{-20}{-32} = .625$$

MAXIMUM HIGHT $= k = s(h) = s(.625) = 204.25$

THE ROCK REACHES THE MAXIMUM ALTITUDE OF 204.25 ft AFTER .625 SECONDS

3. Solve the equation $-3x^2 - 9x + 84 = 0$. (Show your work)

$$\frac{-3x^2-9x+84}{-3} = \frac{0}{-3} \longrightarrow x^2 \boxed{+3}x \boxed{-28} = 0 \longrightarrow$$

SUM PRODUCT $7 \cdot (-4)$

$$\rightarrow (x+7)(x-4) = 0 \; \begin{cases} x+7=0 \rightarrow x=-7 \\ x-4=0 \rightarrow x=4 \end{cases}$$

4. The profit for a product is given by $P(x) = -11x^2 - 1705x + 15950$ (measured in dollars), where x is the number of units produced and sold. How many units give break even for this product?

BREAK EVEN: $P(x) = 0$

$$\frac{-11x^2 - 1705x + 15950}{-11} = \frac{0}{-11} \rightarrow x^2 + 155x - 1450 = 0$$

QUADRATIC FORMULA: $x = \dfrac{-155 \pm \sqrt{155^2 - 4 \cdot 1 \cdot (-1450)}}{2} \rightarrow$

$$\rightarrow x = \frac{-155 \pm 172.7}{2} \; \begin{cases} x = (-155 - 172.7)/2 = -163.85 \\ x = (-155 + 172.7)/2 = 8.85 \end{cases}$$

NOT POSSIBLE A NEGATIVE PRODUCTION

PRODUCING ABOUT 9 UNITS (8.85) ONE BREAKS EVEN.

5. For the nonextreme weather months, Palmetto Electric charges \$7.10 plus 6.747 cents per kilowatt-hour (kWh) for the first 1200 kWh and \$88.06 plus 5.788 cents per kilowatt-hours above 1200.

(a) Write the function that gives the monthly charge in dollars as a function of the kilowatt-hours used.

Y = COST = "FIXED COST" + RATE • QUANTITY ; "RATE" = PRICE ; X kWh CONSUMED

$$Y = \begin{cases} 7.10 + .06747X, & 0 \le X \le 1200 \\ 88.06 + .05788(X-1200), & X > 1200 \end{cases}$$; in DOLLARS

(b) What is the monthly charge if 960 kWh are used?

$Y(960) = 7.10 + .06747 \cdot 960 = 71.8712$

ABOUT \$71.87

(c) What is the monthly charge if 1580 kWh are used?

$Y(1580) = 88.06 + .05788(1580-1200)$

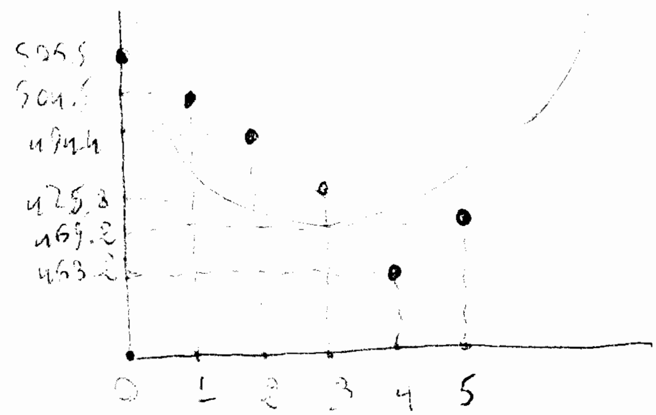
$= 110.0544$

ABOUT \$110.05

6. The following table gives the rate of violent crimes (per 100,000 residents) in the U.S., for years from 2000 to 2005.

Year	2000	2001	2002	2003	2004	2005
Violent Crimes per 100,000	506.5	504.5	494.4	475.8	463.2	469.2

(a) Make a scatter plot of the data, with t equal to the number of years past 2000 and y equal to the rate of violent crimes (per 100,000 residents). (Clearly report the coordinates of the points)



(b) Using your calculator, find the QUADRATIC model which is the best fit for the data. Report your answer to 4 decimal places.

$y = .5357x^2 - 12.0786x + 510.8857$

(NOTICE, A QUARTIC MODEL IS A BETTER FIT)

(c) What rate of violent crimes does the (unrounded) model predict for 2012?

YEAR 2012 IS FOR $X = 12$

$Y(12) = 443.08571$

ABOUT **443.086** VIOLENT CRIMES PER 100,000 RESIDENTS

7. Let $f(x) = x^2 - x + 3$ and $g(x) = 1 - 3x$. Compute $(g \circ f)(x)$ and $f(g(1))$.

$$(g \circ f)(x) = g(f(x)) = 1 - 3(x^2 - x + 3) = 1 - 3x^2 + 3x - 9$$
$$= -3x^2 + 3x - 8$$

$$g(1) = 1 - 3 \cdot 1 = -2 \rightarrow f(g(1)) = f(-2) = (-2)^2 - (-2) + 3$$
$$= 9$$

8. Suppose the total weekly cost for a certain product is $C(x) = 10,000 + 30x + x^2$ dollars and that the total revenue is given by $R(x) = 550x$ dollars, where x is the number of units produced and sold. Write the equation of the function that models the weekly profit for this product, then compute the profit for producing and selling 120 units.

$$P(x) = R(x) - C(x) = 550x - (10000 + 30x + x^2)$$
$$= 550x - 10000 - 30x - x^2 = -10000 + 520x - x^2$$

$$P(120) = -10000 + 520 \cdot 120 - 120^2 = 38000$$

WHEN PRODUCING 120 UNITS, THE PROFIT IS \$38,000.

9. Find the inverse of the function $f(x) = 2 - x^3$ and check your result.

$Y = 2 - X^3$

I) SOLVE FOR X: $\sqrt[3]{x^3} = \sqrt[3]{2 - Y} \rightarrow X = \sqrt[3]{2-Y}$

II) SWAP X & Y: $Y = \sqrt[3]{2-X} = g(x)$

III) CHECK: 1) $f(g(x)) = 2 - \left(\sqrt[3]{2-x}\right)^3 = 2 - (2-x) = 2 - 2 + x = x$ ✓

2) $g(f(x)) = \sqrt[3]{2-(2-x^3)} = \sqrt[3]{2-2+x^3} = \sqrt[3]{x^3} = x$ ✓

10. Let $f(x) = x - 3$ and $g(x) = 4 + 2x$. Compute $(g - f)(x)$ and $(f \cdot g)(x)$.

$(g-f)(x) = g(x) - f(x) = (4+2x) - (x-3)$

$= 4 + 2X - X + 3 = 7 + X$

$(f \cdot g)(x) = f(x)g(x) = (X-3)\cdot(4+2X)$

$= 4X + 2X^2 - 12 - 6X$

$= 2x^2 - 2X - 12$