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Math 102 - Fall 2012 - Test 3

Instructor: Dr. Francesco Strazzullo

Name: KEY

Instructions. If you use graphic methods, sketch the graphs and label significant points, like intersection points or intercepts. Each exercise is worth 10 points, unless otherwise specified. *Always use the appropriate wording and units of measure in your answers (when applicable).*
SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

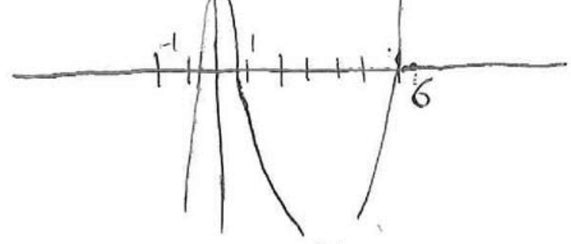
1. Find the rational root(s) of the polynomial $6x^3 - 37x^2 + 4x + 12$.

WITH 2^{ND} + TRACE + ZERO

THREE TIMES WE FIND: $X = 6$,

$X = 2/3 \approx .66667$, $X = -\frac{1}{2} = -.5$.

AT MOST THREE ZEROS AND WE SEE ALL OF THEM.



OTHERWISE: AMONG DIVISORS OF $6 \cdot 12$ WE

CHECK $X = 6$ IS A ZERO → SYNTHETIC OR LONG DIVISION BY $X - 6$

$$\begin{array}{r|rrr|r} & 6 & -37 & 4 & 12 \\ 6 & & 36 & -6 & -12 \\ \hline & 6 & -1 & -2 & 0 \end{array} \checkmark$$

$$\rightarrow 6X^3 - 37X^2 + 4X + 12 = (X-6)\cdot(6X^2 - X - 2)$$

$$\underbrace{6X^2 - X - 2}_{} = 6X^2 + 3X - 4X - 2 = 3X(2X+1) - 2(2X+1)$$

$$= (2X+1)(3X-2) \rightarrow$$

PRODUCT: $6 \cdot (-2) = -12$
SUM: -1
→ $-4, +3$

$$\rightarrow \begin{cases} 2X + 1 = 0 \rightarrow 2X = -1 \rightarrow X = -\frac{1}{2} \\ 3X - 2 = 0 \rightarrow 3X = 2 \rightarrow X = \frac{2}{3} \end{cases}$$

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2. Perform the long division $\dfrac{2x^4 - 11x^2 + 3x - 7}{x^2 + 3}$.

$\dfrac{2x^4}{x^2}$ $\quad$ $\dfrac{-17x^2}{x^2}$

$$
\begin{array}{r|l}
 & 2x^2 - 17 \\
\hline
x^2+3 & 2x^4 - 11x^2 + 3x - 7 \\
 & \underline{2x^4 + 6x^2} \quad \leftarrow 2x^2(x^2+3) \quad \text{SUBTRACT} \\
 & \quad -17x^2 + 3x - 7 \\
 & \quad \underline{-17x^2 \qquad\; - 51} \quad \leftarrow -17(x^2+3) \quad \text{SUBTRACT} \\
 & \qquad\qquad 3x + 44
\end{array}
$$

$$\frac{2x^4 - 11x^2 + 3x - 7}{x^2+3} = 2x^2 - 17 + \frac{3x+44}{x^2+3}$$

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3. During a two-hours eating contest the number y of hot-dogs that Frank can eat each minute (eating rate) is modeled by

$$y = 2x^4 - 17x^2 + 19x + 1,$$

where x is the time in hours since the start of the contest.

(a) How many hot-dogs per minute is Frank eating after 15 minutes?

15 MINUTES = $\frac{15}{60}$ HOURS = .25 HOURS → X = .25 →

→ Y = 4.7 → ALMOST 5 HOT-DOGS PER MINUTE

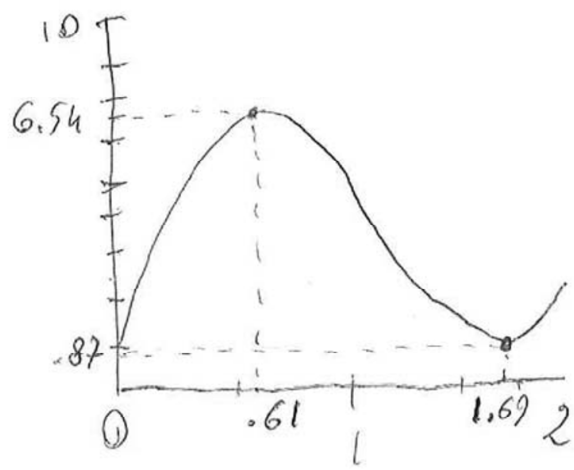
(b) Find out the maximum and the minimum number of hot-dogs Frank can eat in one minute and how far during the contest he reaches these extreme eating rates.

TWO-HOURS CONTEST → $0 \le X \le 2$

I) MAXIMUM: 2^{ND} + TRACE + 4

X = .61 , Y = 6.54 →

→ AFTER ABOUT (.61)60 = 37 MINUTES FRANK IS EATING AT THE MAXIMUM RATE OF ABOUT 6.5 HOT-DOGS PER MINUTE



II) MINIMUM: 2^{ND} + TRACE + 3

X = 1.69 HOURS ≈ 1 HOUR AND 41 MINUTES

Y = .87.

AFTER 1 HOUR AND 41 MINUTES FRANK IS EATING AT THE LOWEST RATE OF LESS THAN ONE HOT-DOG PER MINUTE.

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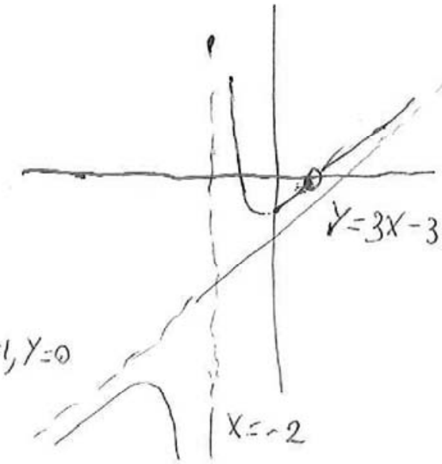
4. For the following rational functions, use algebra to find the domain, then use algebra or technology to find (if any) the asymptotes (vertical, horizontal, or slant). (Each part is worth 10 points)

(a) $f(x) = \dfrac{3x^3 - 6x + 3}{x^2 + x - 2}$

DOMAIN: $X^2+X-2 \neq 0 \rightarrow X \neq 1, -2$

$X^2+X-2 = (X+2)(X-1) \begin{cases} X=1 \\ X=-2 \end{cases}$



V.A.: I) PLUG $X=1 \rightarrow \dfrac{3-6+3}{0} = \dfrac{0}{0} \rightarrow$ LIKELY NOT A V.A.: $f(X) = \dfrac{3(X^2-2X+1)}{(X+2)(X-1)} = \dfrac{3(X-1)^{\cancel{2}}}{(X+2)\cancel{(X-1)}} \rightarrow$ HOLE AT $X=1, Y=0$

II) PLUG $X=-2 \rightarrow 27/0 \rightarrow$ V. ASYMPTOTE: $X=-2$

SLANT-H.A.:

$$\begin{array}{r|l} & 3X-3 \\ X^2+X-2 & 3X^3 - 6X+3 \\ & 3X^3+3X^2-6X \\ \hline & -3X^2+3 \\ & -3X^2-3X+6 \\ \hline & 3X-3 \end{array}$$

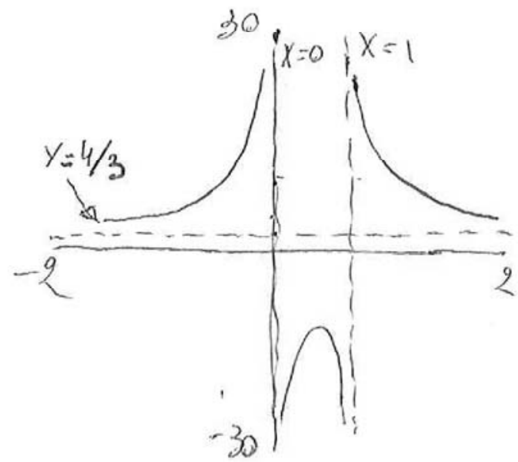
$\rightarrow f(X) = \underbrace{3X-3}_{Y=3X-3} + \dfrac{3X-3}{X^2+X-2}$

$Y = 3X-3$ SLANT ASYMPTOTE

(b) $f(x) = \dfrac{4x^2 + x + 1}{3x^2 - x}$

DOMAIN: $3X^2 - X \neq 0 \quad X \neq 0, 1$

$3X^2 - X = 3X(X-1) \begin{cases} X=0 \\ X=1 \end{cases}$



V.A.: I) PLUG $X=0 \rightarrow \dfrac{1}{0} \rightarrow$ V.A. $X=0$

II) PLUG $X=1 \rightarrow \dfrac{6}{0} \rightarrow$ V.A. $X=1$

SLANT-H.A.: NUM AND DENOM. SAME DEGREE $\rightarrow$ H.A. $Y = \dfrac{4X^2}{3X^2} = \dfrac{4}{3}$

OTHERWISE LONG DIVISION

$$\begin{array}{r|l} & 4/3 \\ 3X^2-X & 4X^2+X+1 \\ & 4X^2 - \frac{4}{3}X \\ \hline & \frac{7}{3}X+1 \end{array}$$

$\rightarrow$ H.A. $Y = \dfrac{4}{3}$

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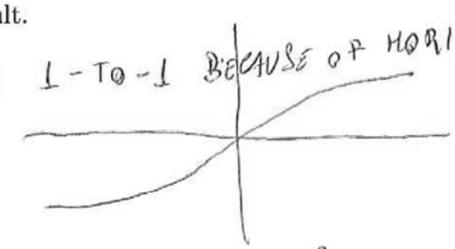
5. Let $g(x) = 2x + 3$ and $f(x) = x^2 - 2$. Compute $(f \circ g)(x)$ and $(f \cdot g)(-1)$.

$$(f \circ g)(x) = f(g(x)) = (2x+3)^2 - 2 = (2x)^2 + 2 \cdot 3 \cdot 2x + 3^2 - 2$$
$$= 4x^2 + 12x + 7$$

$$(f \cdot g)(-1) = f(-1) \cdot g(-1) = (-1)(1) = -1$$
$$f(-1) = (-1)^2 - 2 = 1 - 2 = -1,$$
$$g(-1) = 2(-1) + 3 = -2 + 3 = 1,$$

6. Find the inverse of the function $f(x) = \dfrac{\sqrt[3]{5x}}{4}$ and check your result.

IT IS 1-TO-1 BECAUSE OF HORIZ. LINE TEST



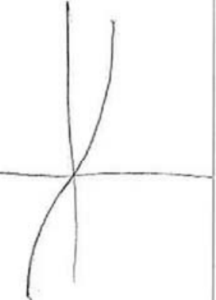
PLUG IN CALCULATOR: $Y_1 = \frac{1}{4}(5X)^\wedge(1/3)$

•) PROCEDURE FOR $f^{-1}(x)$:

1) SOLVE FOR X: $Y = \dfrac{\sqrt[3]{5X}}{4} \rightarrow 4Y = \sqrt[3]{5X} \rightarrow (4Y)^3 = 5X$

$$\rightarrow X = \frac{4^3 Y^3}{5} \rightarrow X = \frac{64}{5} Y^3$$

2) SWAP X AND Y: $Y = \dfrac{64}{5} X^3 \rightarrow f^{-1}(x) = \dfrac{64}{5} x^3$.



••) CHECK: I) $f(f^{-1}(x)) = \dfrac{\sqrt[3]{\cancel{5}\left(\frac{64}{\cancel{5}} x^3\right)}}{4} = \dfrac{\sqrt[3]{64} \cdot \sqrt[3]{x^3}}{4} = \dfrac{4x}{4} = x$ ✓

II) $f^{-1}(f(x)) = \dfrac{64}{5}\left(\dfrac{\sqrt[3]{5x}}{4}\right)^3 = \dfrac{64}{5} \dfrac{(\sqrt[3]{5x})^3}{4^3} = \dfrac{64}{5} \dfrac{5x}{64} = x$ ✓

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