MAT 102 - Exam1 - Spring 2015

Instructor: Dr. Francesco Strazzullo **Name** Key

Instructions. Each problem is worth 10 points. Remember to check your solutions and "box" them reduced to lowest terms or with decimal numbers rounded to two decimal places. You might need some of the following formulas:

- $A = kB$ • $A = {}^{k}/_{B}$ • $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ • $(A \pm B)^2 = A^2 \pm 2AB + B^2$
- $A^3 \pm B^3 = (A \pm B)(A^2 \mp AB + B^2)$ • $A^2 - B^2 = (A - B)(A + B)$
- $h = -\frac{1}{2}gt^2 + v_0 t + h_0$, with $g = 32\frac{ft}{sec^2} \approx 9.8\frac{m}{sec^2}$

SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

1. Simplify the following radical expression

$\sqrt{-80x^7}$

$\sqrt{-80x^7} = \sqrt{16 \cdot 5 \cdot (-1) \cdot x^6 \cdot x} = \sqrt{16} \cdot \sqrt{-1} \cdot \sqrt{x^6} \cdot \sqrt{5x}$

$= 4i x^3 \sqrt{5x}$

2. Solve the following quadratic equation.

$12x^2 + 5x = 2$

$-2 \quad -2$

$12x^2 + 5x - 2 = 0$

$12x^2 + 8x - 3x - 2 = 0$

$4x(3x+2) - (3x+2) = 0$

$(3x+2)(4x-1) = 0$

PRODUCT $= 12(-2) = -24$

PAIRS: $\pm(-1, 24), \pm(-2, 12), \pm(-3, 8), \pm(-4, 6)$

SUM $= 5 = 8 - 3$ ✓

$3x+2 = 0 \Rightarrow 3x = -2 \Rightarrow x = -\frac{2}{3}$

$4x-1 = 0 \Rightarrow 4x = 1 \Rightarrow x = \frac{1}{4}$

20

3. Two brothers, Tom and Brian, each inherit $24000. Tom invests his inheritance in a savings account with an annual return of 2.5%, while Brian invests his inheritance in a CD paying 4.5% annually. How much more money than Tom does Brian have after 1 year?

SIMPLE INTEREST I DIRECTLY PROPORTIONAL TO INVESTMENT P: $I = rP$
WITH r YEARLY INTEREST RATE. $P = \$24000$ FOR BOTH.

TOM: $r = 2.5\% = \frac{2.5}{100} = 0.025 \Rightarrow I = 24000(0.025) = 600$

BRIAN: $r = 4.5\% = \frac{4.5}{100} = 0.045 \Rightarrow I = 24000(.045) = 1080$

DIFFERENCE $= 1080 - 600 = 480$ DOLLARS

NOTE: SAME PRINCIPAL GIVES A DIFFERENCE OF INTERESTS GIVEN BY THE DIFFERENCE OF RATES: $4.5\% - 2.5\% = 2\% \Rightarrow$ DIFFERENCE $= 24000(.02)$ ✓

4. Consider the following absolute value inequality.

$$|4x + 3| \leq 6$$

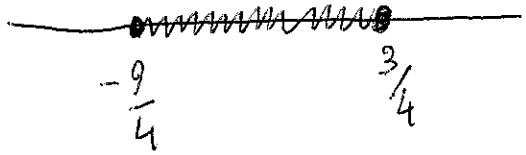
(a) Describe the solution set using interval notation, and (b) graph the solution set.

$$|4x+3| \leq 6 \iff \begin{cases} 4x+3 \leq 6 \Rightarrow 4x \leq 3 \Rightarrow x \leq \frac{3}{4} \\ \text{AND} \\ 4x+3 \geq -6 \Rightarrow 4x \geq -9 \Rightarrow x \geq -\frac{9}{4} \end{cases} \Rightarrow$$

a)

$$\Rightarrow -\frac{9}{4} \leq x \leq \frac{3}{4} \quad \text{OR} \quad x \text{ IN } \left[-\frac{9}{4}, \frac{3}{4}\right]$$

b)



20

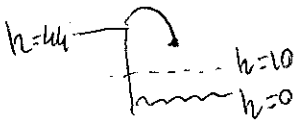
5. A rock is thrown upward with a speed of 24 meters per second from the top of a 44 meter high cliff, and it misses the cliff on the way back down. When will the rock be 10 meters from the water, below?

FREE FALLING OBJECT: $h = -\frac{1}{2}gt^2 + v_0 t + h_0$

USED METERS: $g = 9.8$; $v_0 = 24$; $h_0 = 44$ CHOOSING $h=0$ AT SEA LEVEL.

ASKED FOR THE TIME WHEN $h = 10$: $-\frac{1}{2}(9.8)t^2 + 24t + 44 = 10$

$-4.9t^2 + 24t + 34 = 0$



$$t = \frac{-24 \pm \sqrt{24^2 - 4(-4.9)(34)}}{2(-4.9)} = \frac{-24 \pm \sqrt{1242.4}}{-9.8} \Rightarrow$$

$$\Rightarrow t \approx \frac{-24 \pm 35.25}{-9.8} \begin{cases} t \approx -1.12 \text{ REJECT} \\ t \approx 6.02 \checkmark \end{cases}$$

AT 10 METERS ABOVE WATER AFTER ABOUT 6 SECONDS

6. Solve the following rational equation.

$$\frac{x}{x-3} - \frac{8}{x+6} = \frac{x^2}{x^2+3x-18}$$

$x^2+3x-18 = (x+6)(x-3)$ THIS IS ALSO THE LCD.

RESTRICTED VALUES: $x-3=0 \Rightarrow x=3$

$x+6=0 \Rightarrow x=-6$.

MULTIPLY BY LCD: $(x+6)(x-3)\cdot\frac{x}{x-3} - (x+6)(x-3)\cdot\frac{8}{x+6} = \frac{x^2}{(x+6)(x-3)}\cdot(x+6)(x-3)$

$\Rightarrow (x+6)x - 8(x-3) = x^2 \Rightarrow x^2 + 6x - 8x + 24 - x^2 = 0 \Rightarrow$

$\Rightarrow -2x + 24 = 0 \Rightarrow -2x = -24 \Rightarrow x = \frac{-24}{-2} = 12$ NOT RESTRICTED

$\boxed{x = 12}$

7. You try to fill your cracked 12 cc cup with water. The drinking fountain fills the cup at a rate of 2 cc per second, while from the crack water drips at a rate of 0.6 cc per second. How long would it take to fill your cup? (Set up a rational equation and solve, rounding to one decimal place)

JOB = "FILL THE 12 cc CUP" ⇒ CRACK (DRIPPING) HAS A NEGATIVE RATE

NOTE: VOLUME = TIME · RATE

$V = T \cdot R \Rightarrow T = \frac{V}{R}$

	TIME (SECONDS)	WORK RATE
FOUNTAIN	$\frac{12}{2} = 6$	$\frac{1}{6}$
CRACK	$\frac{12}{0.6} = 20$	$-\frac{1}{20}$
TOGETHER	X	$\frac{1}{X}$

$\frac{1}{6} - \frac{1}{20} = \frac{1}{X}$

$\frac{7}{60} = \frac{1}{X} \Rightarrow X = \frac{60}{7} \approx 8.6$

IT WILL TAKE ABOUT 8.6 SECONDS (INSTEAD OF THE 6 FOR AN INTACT CUP)

8. Solve the following radical equation.

$$\sqrt{4x+17} + 9 = x + 8$$
$$-9 \quad -9$$

$\left(\sqrt{4x+17}\right)^2 = (x-1)^2 \Rightarrow 4x + 17 = x^2 - 2x + 1 \Rightarrow$

$-4x \quad -17 \qquad -4x \quad -17$

$\Rightarrow x^2 - 6x - 16 = 0 \Rightarrow (x-8)(x+2) = 0$

$x - 8 = 0 \Rightarrow x = 8$

$x + 2 = 0 \Rightarrow x = -2$

CHECK:

- $x = 8$: $LHS = \sqrt{4(8)+17} + 9 = \sqrt{49} + 9 = 7 + 9 = 16$ ✓
  $RHS = 8 + 8 = 16$
- $x = -2$: $LHS = \sqrt{4(-2)+17} + 9 = \sqrt{9} + 9 = 3 + 9 = 12$ NOT THE SAME
  $RHS = -2 + 8 = -6$

ONLY ONE SOLUTION: $\boxed{x = 8}$