MAT 121 – Exam4 – Spring 2016

Instructor: Dr. Francesco Strazzullo **Name**________________________________

Instructions. Complete the following exercises. Each exercise is worth 10 points. If you need to approximate then **round to 3 decimal places**, unless otherwise specified. You can use your own cheat sheet after I approve it, or the one on Eagleweb. You can also use a graphing tool and/or a computer algebra system like GeoGebra. When solving a problem graphically sketch the graph you used.
SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

1. Solve for the remaining angles and side of the one triangle that can be created. Round to the nearest hundredth:

$$B = 90°, b = 5.5, c = 2$$

RIGHT TRIANGLE WITH HYPOTENUSE b : $a = \sqrt{b^2 - c^2} =$
$= \sqrt{(5.5)^2 - 2^2} = \frac{1}{2}\sqrt{105}$

$\sin C = \frac{c}{b} \sin b = \frac{c}{b} = \frac{2}{5.5} = \frac{4}{11} \Rightarrow C = \sin^{-1}\left(\frac{4}{11}\right) \approx 21.32°$

$A = 90° - 21.32° = 68.68°$

2. Nick wants to build an herb garden in the backyard but is only using the area beyond the path running through the yard. How large would the garden be if it is triangular-shaped with sides of length 10 feet, 7 feet, and 9 feet? Round to the nearest hundredth.

Area of a Triangle (Heron's Formula)

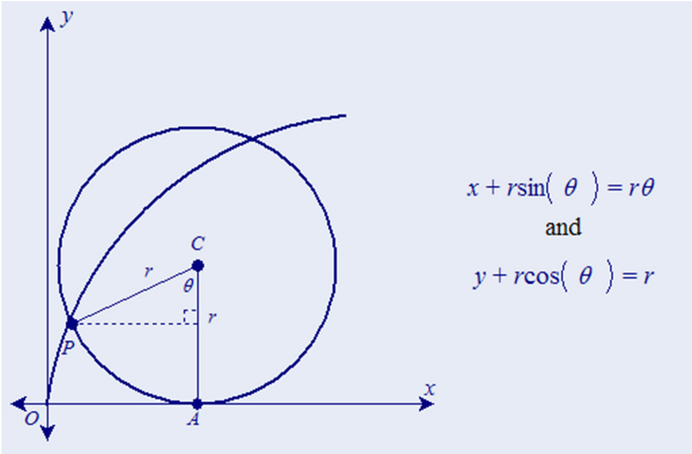
Given a triangle with sides a, b and c, let $s = \frac{a+b+c}{2}$. Then

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$a = 10$
$b = 7$
$c = 9$

$s = \frac{10+7+9}{2} = 13 \Rightarrow A = \sqrt{13 \cdot 3 \cdot 6 \cdot 4} = 6\sqrt{26} \approx 30.59 \text{ FT}^2$

3. A wheel with a radius of 13 inches rolls along a flat surface in a straight line. There is a fixed point P that initially lies at the point $(0,0)$. Find parametric equations in terms of θ describing the cycloid traced out by P.



$r = 13$

$$\begin{cases} x = 13(\theta - \sin\theta) \\ y = 13(1 - \cos\theta) \end{cases}$$

4. Consider the following parametric equations:

$$x = \sin(\theta) + 2 \text{ and } y = 2\sin(\theta) - 2$$

(a) Eliminate the parameter θ, by writing your answer in simplest form solved for y.
(b) Determine the domain and range of the equation obtained by eliminating the parameter. Please write your answer in interval notation.

$\sin\theta = x - 2 \Rightarrow y = 2(x-2) - 2 = 2x - 6$

$-1 \le \sin \le 1 \Rightarrow -1 \le x - 2 \le 1 \Rightarrow 1 \le x \le 3$

$\Downarrow$

$2(-1) - 2 \le y \le 2(1) - 2 \Rightarrow -4 \le y \le 0$

DOMAIN $= [1, 3]$, RANGE $= [-4, 0]$

5. Use any convenient method to solve the following system of equations. Indicate the number of solutions to this system. State the solution, if one exists, as an ordered triple, and if there are infinitely many solutions, express the solution set in terms of one of the variables. Leave all fractional answers in fraction form.

$$\begin{cases} -2x + y + 3z = 13 \\ x + 3y - 4z = -15 \\ 3x + 2y - 7z = -28 \end{cases}$$

A

$$\to \begin{pmatrix} -2 & 1 & 3 & 13 \\ 1 & 3 & -4 & -15 \\ 3 & 2 & -7 & -28 \end{pmatrix}$$

ReducedRowEchelonForm[A]

$$\to \begin{pmatrix} 1 & 0 & -\frac{13}{7} & -\frac{54}{7} \\ 0 & 1 & -\frac{5}{7} & -\frac{17}{7} \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{cases} x - \frac{13}{7} z = -54/7 \\ y - \frac{5}{7} z = -\frac{17}{7} \end{cases} \to \begin{cases} x = \frac{13z - 54}{7} \\ y = \frac{5z - 17}{7} \end{cases}$$

z IS FREE

SOLUTION $= \left\{ \left(\frac{13z-54}{7}, \frac{5z-17}{7}, z \right) \mid z \text{ IN } \mathbb{R} \right\}$

6. Use Gauss-Jordan elimination to solve the following system of equations.

$$\begin{cases} 4w - y = 8 \\ w - x + z = 8 \\ -3w - 6y - z = -35 \\ x + 6y = 21 \end{cases}$$

Indicate the number of solutions to this system. State the solution, if one exists, as an ordered quadruple, and if there are infinitely many solutions, express the solution set in terms of one of the variables.

B

$$\to \begin{array}{c} \begin{array}{ccccc} w & x & y & z & \end{array} \\ \begin{pmatrix} 4 & 0 & -1 & 0 & 8 \\ 1 & -1 & 0 & 1 & 8 \\ -3 & 0 & -6 & -1 & -35 \\ 0 & 1 & 6 & 0 & 21 \end{pmatrix} \end{array}$$

ReducedRowEchelonForm[B]

$$\to \begin{pmatrix} 1 & 0 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 & -3 \\ 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 1 & 2 \end{pmatrix} \to \begin{cases} w = 3 \\ x = -3 \\ y = 4 \\ z = 2 \end{cases}$$

SOLUTION $(3, -3, 4, 2)$.

7. In the table below it is reported the time of the day (astronomic time) corresponding to the sunrise in Atlanta last year.

Month, t	Sunrise time in Atlanta on the 1st of the month	Sunrise T minutes after 6:00 AM
1	8:43 AM	163
2	8:34 AM	154
3	8:06 AM	126
4	7:25 AM	85
5	6:49 AM	49
6	6:28 AM	28
7	6:31 AM	31
8	6:50 AM	50
9	7:12 AM	72
10	7:32 AM	92
11	7:57 AM	117
12	8:24 AM	144

(a) Use the sine regression feature of a graphing utility to find the sine model that best fits these data. Report the models and the corresponding coefficient of determination (rounding to 4 decimal places).

(b) What is the amplitude and the period of this model? (Use the units of measure as well)

(c) When is the sun in Atlanta rising the earliest and the latest? (Round to the hundredths)

(d) What is the mean rising time over one year?

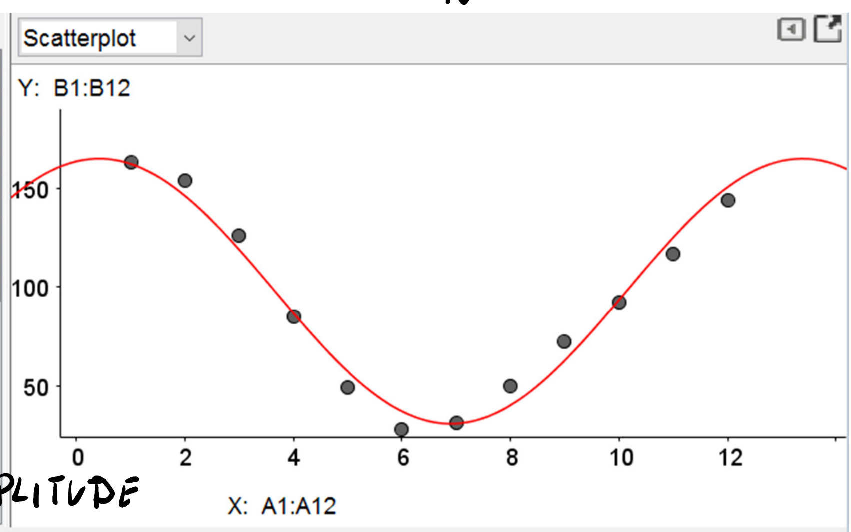
(a) IN GRAPH BELOW

(b) AMPLITUDE = 67 MINUTES

PERIOD = $\frac{2\pi}{.4848} \approx 13$ MONTHS

Statistics

MeanX	6.5
MeanY	92.5833
Sx	3.6056
Sy	47.9061
r	-0.2208
ρ	-0.2238
Sxx	143
Syy	25244.9167
Sxy	-419.5
R^2	0.9779
SSE	558.7623



MEAN

AMPLITUDE

Regression Model

Sin

$y = 97.8594 + 67.1197 \sin(0.4848x + 1.368)$

(c) BY GRAPH, OR ALGEBRA:

EARLIEST → MIN → $\sin(\theta) = -1$ → $.4848X + 1.368 = \frac{3}{2}\pi$ →

→ $X \approx 6.9$ MONTHS (ALMOST JULY)

LATEST → MAX → $\sin\theta = 1$ → $.4848X + 1.368 = \frac{\pi}{2}$ → $X \approx .42$ (MID JANUARY)

(d) MEAN (OF MODEL) ≈ 98 MINUTES AFTER 6 AM, THEN TIME IS 7:38 AM.