MAT 102 – Exam2 - Fall 2013

Instructor: Dr. Francesco Strazzullo **Name** KEY

I certify that I did not receive third party help in *completing* this test (sign) _____

Instructions. Technology is allowed on this exam. Each problem is worth 10 points. If you use formulas or properties from our book, make a reference. When using technology describe which commands (or keys typed) you used or print out and attach your worksheet.
SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

1. Write the **slope-intercept form** of the equation for the line that passes through the point $(7, 8)$ and has a slope of 2.

$y = mx + b$, $m = 2$ $\rightarrow$ $y = 2x + b$ $\rightarrow$ PLUG POINT $x = 7$, $y = 8$ $\rightarrow$ $8 = 2(7) + b \Rightarrow b = -6$ (-14 -14)

$$\boxed{y = 2x - 6}$$

2. Consider the following equation of a line.

$$x + 7y = 5y - 5$$

a. Rewrite this equation in slope-intercept form. Reduce all fractions to lowest terms.
b. Find the equation, in slope-intercept form, for the line which is **perpendicular** to this line and passes through the point $(9, -3)$. Reduce all fractions to lowest terms.

(a) SLOPE-INTERCEPT: $y = mx + b$. $x + 7y = 5y - 5 \Rightarrow$ ($-x$ $-5y$ $-5y$ $-x$)

$\Rightarrow \frac{2y}{2} = \frac{-x - 5}{2} \Rightarrow \boxed{y = -\frac{1}{2}x - \frac{5}{2}}$

(b) PERPENDICULAR to (a): SLOPE $= -\frac{1}{m} = -\frac{1}{-\frac{1}{2}} = 2 \Rightarrow y = 2x + b$

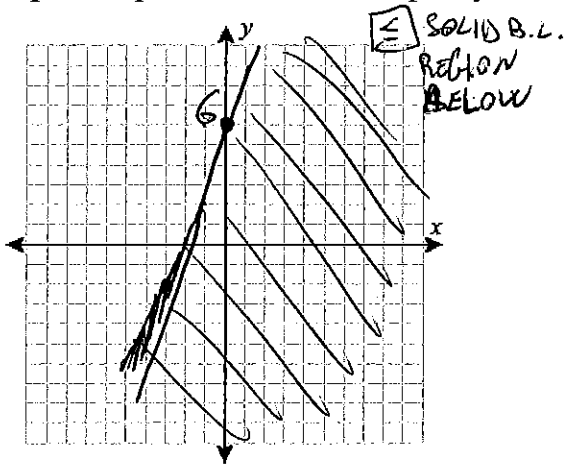
PLUG $x = 9$, $y = -3 \Rightarrow -3 = 2(9) + b \Rightarrow b = -21$ (-18 -18)

$$\boxed{y = 2x - 21}$$

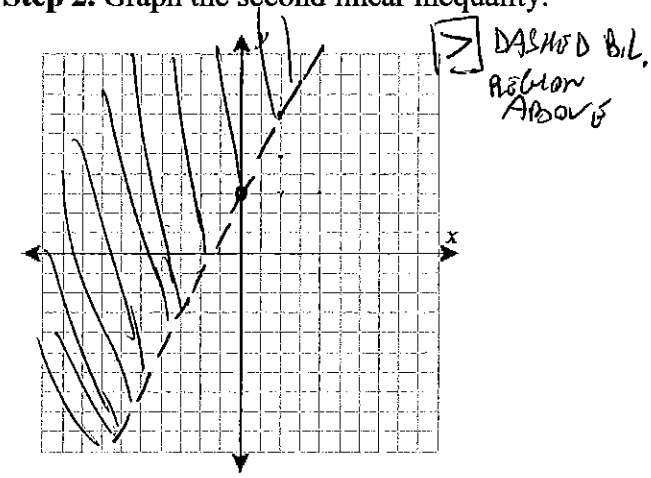
3. Solve the system of two linear inequalities graphically.

$$y \leq 4x + 6 \text{ and } y > 2x + 3$$

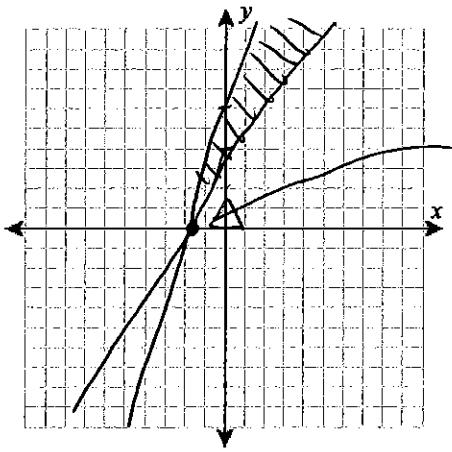
Step 1. Graph the first linear inequality.



Step 2. Graph the second linear inequality.



Step 3. Graph both inequalities and highlight the solution set of this system of linear inequalities. Also, mark your selection A or B.



A) the union of the individual solution sets

~~B)~~ the intersection of the individual solution sets

NOTE: CORNER POINT $\left(-\frac{3}{2}, 0\right)$

$$\begin{cases} Y = 4x + 6 \\ Y = 2x + 3 \end{cases} \Rightarrow 4x + 6 = 2x + 3$$

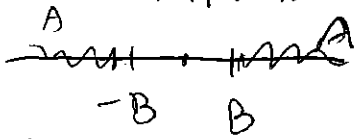
$$\Rightarrow 2x = -3 \Rightarrow x = -\frac{3}{2}$$

$$\Rightarrow y = 2\left(-\frac{3}{2}\right) + 3 = 0$$

4. Graph the solution set of the following linear inequality:

$|5x + 4y| > 19$

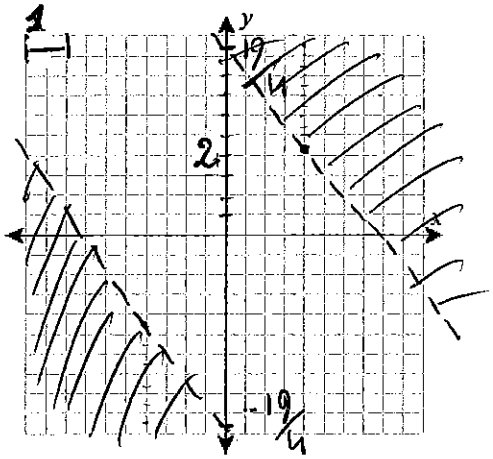
$|A| > B$



$5x+4y > 19 \Rightarrow 4y > -5x+19 \Rightarrow y > -\frac{5}{4}x + \frac{19}{4}$

OR (UNION)

$5x+4y < -19 \Rightarrow y < -\frac{5}{4}x - \frac{19}{4}$



5. Find the standard form of the equation for the circle described below.

Center $(-1, -6)$ and radius 2

$$(x-x_0)^2 + (y-y_0)^2 = R^2$$

$$(x-(-1))^2 + (y-(-6))^2 = 2^2$$

$$\boxed{(x+1)^2 + (y+6)^2 = 4}$$

6. Consider the following quadratic function.

$$g(x) = (x - 5)^2 + 2$$

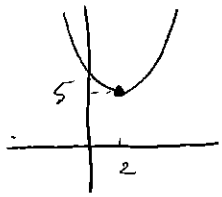
a. Find the vertex of this function.
b. Determine the x-intercept(s).

(a) IT IS IN VERTEX FORM: $y = a(x-h)^2 + k \Rightarrow (h, k) = (5, 2)$

(b) X-INTERCEPTS $\Rightarrow$ REAL SOLUT. OF $(x-5)^2 + 2 = 0 \Rightarrow$

$\Rightarrow \sqrt{(x-5)^2} = \sqrt{-2} \Rightarrow$ COMPLEX SOLUTIONS $\Rightarrow$ NO X-INTERCEPTS

NOTE: VERTEX ABOVE X-AXIS AND LEADING COEFF. POSITIVE $\Rightarrow$ PARABOLA UPWARD NOT INTERSECTING THE X-AXIS



7. Graph the following function.

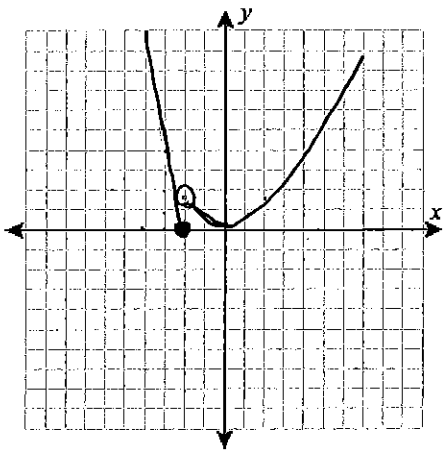
$$t(x) = \begin{cases} -5x - 10 & \text{if } x \leq -2 \\ \frac{2}{5}x^2 & \text{if } x > -2 \end{cases}$$

$\rightarrow$ STRAIGHT LINE ON LEFT

$\rightarrow$ PARABOLA ON RIGHT

$t(-2) = -5(-2) - 10 = 0$

PARABOLA AT $x = -2$: $y = \frac{2}{5}(-2)^2 = \frac{8}{5} \approx 1.6$



8. The volume of a gas in a container varies inversely as the pressure on the gas. If a gas has a volume of 171 cubic inches under a pressure of 3 pounds per square inch, what will be its volume if the pressure is increased to 4 pounds per square inch? (Round off your answer to the nearest integer.)

INVERSE VARIATION: $A = \frac{K}{B}$

HERE P = "PRESSURE" AND V = "VOLUME" $\Rightarrow V = \frac{K}{P} \Rightarrow \frac{K}{3} = 171 \Rightarrow$

GIVEN DATA: $V = 171$ WHEN $P = 3$

$\Rightarrow K = 513 \Rightarrow V = \frac{513}{P}$

"INCREASED TO 4" $\Rightarrow P = 4 \Rightarrow V = \frac{513}{4} \approx 128.2 \approx 128$

$V = 128$ CUBIC INCHES

9. For $f(x) = \frac{1}{x}$ and $g(x) = \frac{x-5}{2}$

Step 1. Determine the formula and domain for $(f \circ g)(x)$. Write your answer in simplest form. Round your answer to two decimal places, if necessary.

$f(g(x)) = \frac{1}{\frac{(x-5)}{2}} = \frac{2}{x-5}$

DOMAIN: $x - 5 \neq 0 \Rightarrow x \neq 5$ OR $(-\infty, 5) \cup (5, +\infty)$

Step 2. Compute $(g \circ f)(-1)$.

$(g \circ f)(-1) = g(f(-1)) = g\left(\frac{1}{-1}\right) = g(-1) = \frac{-1-5}{2} = -3$

10. Consider the function

$$P(x) = 4\sqrt[3]{x} + 4$$

a. Find a formula for the inverse of the given function, if possible. If the function does not have an inverse, write "does not have an inverse function".

b. Check your answer in **a.** by computing the compositions.

NOTE: $P(x)$ IS ONE-TO-ONE.

(a) 1) SWAP x AND y: $x = 4\sqrt[3]{y} + 4$

2) SOLVE FOR y: $4\sqrt[3]{y} = x - 4 \Rightarrow \sqrt[3]{y} = \frac{x-4}{4} \Rightarrow y = \left(\frac{x-4}{4}\right)^3$

$$P^{-1}(x) = \left(\frac{x-4}{4}\right)^3$$

(b) CHECK:

I) $P(P^{-1}(x)) = 4\sqrt[3]{\left(\frac{x-4}{4}\right)^3} + 4 = 4\left(\frac{x-4}{4}\right) + 4 = x$ ✓

II) $P^{-1}(P(x)) = \left(\frac{(4\sqrt[3]{x}+4)-4}{4}\right)^3 = \left(\frac{4\sqrt[3]{x}+4-4}{4}\right)^3 = \left(\sqrt[3]{x}\right)^3 = x$ ✓