**MAT 121 - Exam1 – Spring 2014**

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**Instructions.** Complete 7 out of the following 11 exercises, as indicated. Exercise 12 is for extra points. Each exercise is worth 10 points. You can use a graphing tool and/or a computer algebra system like GeoGebra. When solving a problem graphically sketch the graph you used.
**SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).**

**Complete 3 of the exercises 1-4**

1. The simple interest on an investment is directly proportional to the amount of the investment. By investing $8750 in a certain certificate of deposit, you obtained an interest payment of $420.00 after 1 year. Determine a mathematical model that gives the interest, *I*, for this CD after 1 year in terms of the amount invested, *P*.

SIMPLE INVESTMENT INTEREST = $I$

AMOUNT INVESTED = PRINCIPAL = $P$

"PHRASE #1": $\frac{I}{P} = K$ (CONSTANT)

DATA: $\frac{420}{8750} = K$ THEN $K = .048$ (THAT IS A 4.8% RATE)

$\frac{I}{P} = K \Rightarrow I = KP$ THEN

LINEAR MODEL: $I = .048P$

2. Carl's Tractor Service purchases a used brush mower for $650. The machine has a useful life of 10 years after which time another one will have to be purchased. Assume depreciation of the machine is linear. Write a linear equation giving the value $V$ of the used brush mower during the 10 years it will be in use.

"TIME IN YEARS" $= t$ ; "MOWER VALUE" $= V$ (IN DOLLARS)

INITIAL VALUE: $V = 650$ FOR $t = 0$. → $b = 650$.

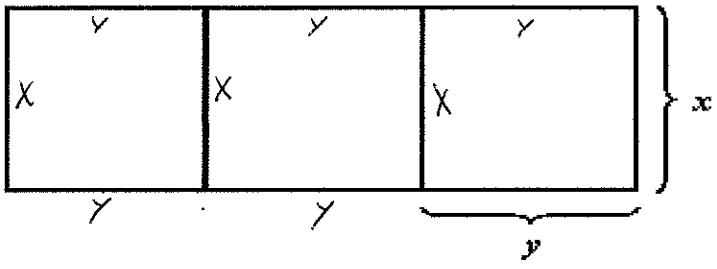
USEFUL LIFE: $V = 0$ FOR $t = 10$

LINEAR DEPRECIATION: $V = mt + b$

DEPRECIATION RATE $= m = \frac{0 - 650}{10 - 0} = -65$ \$/YEAR

**LINEAR MODEL: $V = -65t + 650$**

3. A farmer has 264 feet of fencing and wants to build three identical pens for his prize-winning pigs. The pens will be arranged as shown. Determine the dimensions of a pen that will maximize its area.




AREA OF ONE PEN: $A = XY$

TOTAL PERIMETER: $P = 4X + 6Y$

DATA: $P = 264$ THEN $4X + 6Y = 264 \Rightarrow Y = \frac{264}{6} - \frac{4}{6}X$

OR $Y = 44 - \frac{2}{3}X$. PLUG IN AREA:

$A = X\left(44 - \frac{2}{3}X\right) = 44X - \frac{2}{3}X^2$ OR $A = -\frac{2}{3}x^2 + 44X$

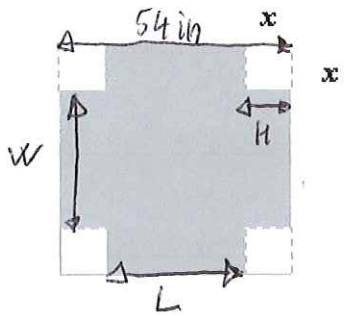
BY ALGEBRA (YOU COULD USE A GRAPH). MAX AT VERTEX: $X = -\frac{b}{2a}$

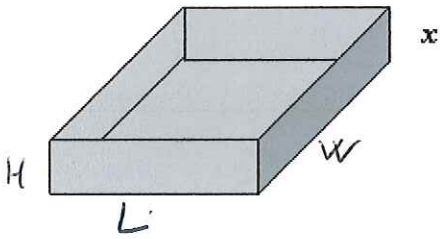
OR $X = \frac{-44}{2\left(-\frac{2}{3}\right)} = 33$ FT. THEN:

$Y = 44 - \frac{2}{3}(33) = 22$ FT.

**PEN: 22' × 33'**

4. An open box is to be made from a square piece of cardboard, 54 inches on a side, by cutting equal squares with sides of length $x$ from the corners and turning up the sides (see figure below). After determining the function $V$, in terms of $x$, that represents the volume of the box, use a graphing utility to estimate the dimensions that will maximize its volume.



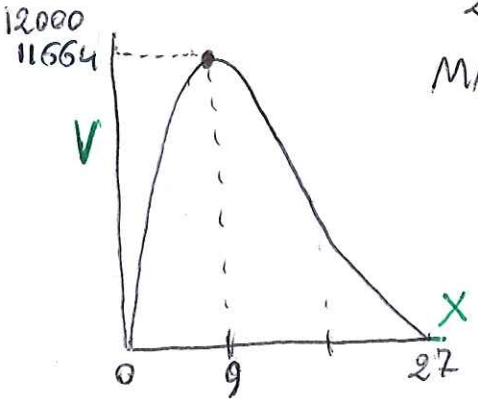

$V = L \cdot W \cdot H$

$H = x$

$L = W = 54 - 2x$

THEN: $V = (54-2x)^2 \cdot x \Rightarrow V = 4x^3 - 216x^2 + 2916x$

NOTE $0 \le x \le \frac{54}{2}$ (THAT IS $0 \le x \le 27$) BECAUSE WE CUT EACH CORNER OUT.

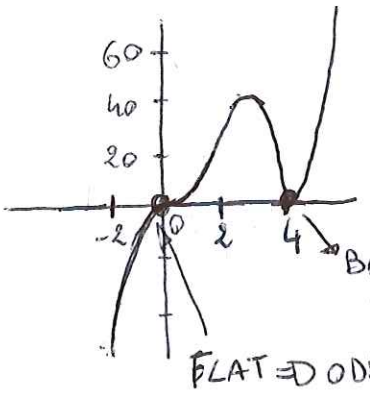



MAX = (9, 11664)

$\boxed{x = 9 \text{ in}}$ GIVES MAX VOLUME ($V = 11664 \text{ in}^3$)

**Complete 2 of the exercises 5-7**

5. Using a graphing utility, graph $f(x) = x^5 - 8x^4 + 16x^3$ and approximate the zeros and their multiplicity.




ZEROS ARE $x = 0$ AND $x = 4$

MULTIPLICITIES ARE BIGGER THAN 0 AND SMALLER THAN DEGREE, 5.

BOUNCING $\Rightarrow$ EVEN MULTIPLICITY, 2 OR 4

FLAT $\Rightarrow$ ODD MULTIPLICITY $> 1 \Rightarrow$ 3, 5

] THE SUM OF

MULTIPLICITIES MUST BE $\le 5$ (DEGREE) $\Rightarrow$ $\boxed{3 \text{ AT } x = 0 \text{ AND } 2 \text{ AT } x = 4}$

BY ALGEBRA: $f(x) = x^3(x^2 - 8x + 16) = x^3(x-4)^2$ ✓

6. Using the factors $(x-5)$ and $(x-2)$, find the remaining factor(s) of $f(x)=x^3-6x^2+3x+10$ and write the polynomial in fully factored form.

OR GRAPH TO FIND ROOT,

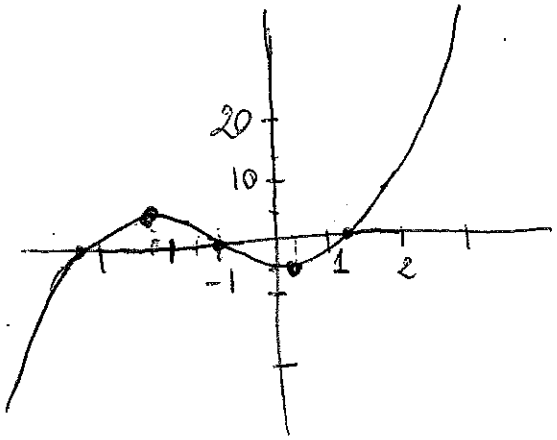
YOU COULD USE A CAS TO FACTOR, OR ALGEBRA. $f(x)$ IS DIVIDED BY $(x-5)(x-2)=x^2-7x+10$. LONG DIVISION:

$$x+1$$
$$x^2-7x+10 \;\overline{)\; x^3-6x^2+3x+10}$$
$$x^3-7x^2+10x$$
$$x^2-7x+10$$
$$x^2-7x+10$$
$$0$$

$$f(x)=(x-5)(x-2)(x+1)$$

7. Use a graphing utility to graph the function and approximate (to two decimal places) any relative minimum or relative maximum values.

$f(x)=x^3+3x^2-2x-4$




RELATIVE:

MAX $=(-2.29,\ 4.30)$

MIN $=(0.29,\ -4.30)$

**Complete 1 of the exercises 8-9**

8. Simplify the rational expression, $\frac{x^4+3x^3-3x^2-11x-6}{x^2+2x+1}$, by using long division or synthetic division.

MUST BE USED!

HERE A CAS CAN BE USED TO CHECK.

$$x^2+2x+1 \;\overline{)\; x^4+3x^3-3x^2-11x-6}$$

$x^2$

$x^4+2x^3+x^2$ SUBTR.

$x^3-4x^2-11x-6$

$x^3+2x^2+x$ SUBTR.

$-6x^2-12x-6$

$-6x^2-12x-6$ SUBTR.

$0$ ← NO REMAINDER

$= x^2+x-6$

$= (x+3)(x-2)$

9. Use long division to divide.

$f(x) = \left(x^4-3x^2+5\right) \div \left(x^2-4x-3\right)$ (YOU COULD USE A CAS ONLY TO CHECK)

$$x^2-4x-3 \;\overline{)\; x^4-3x^2+5}$$

$x^2$

$x^4-4x^3-3x^2$ SUBTR.

$4x^3+5$

$4x^3-16x^2-12x$ SUBTR.

$+16x^2+12x+5$

$-16x^2+64x+48$ ADD

$+76x+53$

$$f(x) = x^2+4x+16+\frac{76x+53}{x^2-4x-3}$$

**Complete 1 of the exercises 10-11**

10. Show algebraically that $f$ and $g$ are inverse functions. → (i) $f(g(x)) = x$ AND (ii) $g(f(x)) = x$

$f(x) = -6x - 8$ $\qquad$ $g(x) = -\frac{x+8}{6}$

(i) $f(g(x)) = -6\left(-\frac{x+8}{6}\right) - 8 = x + 8 - 8 = x$ ✓

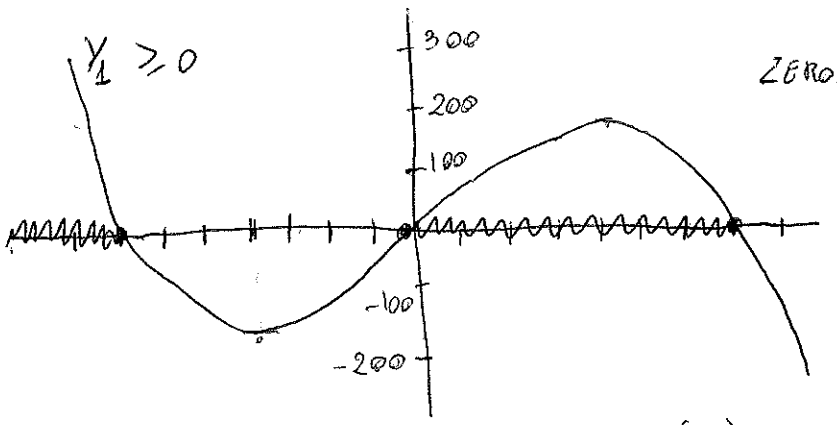
(ii) $g(f(x)) = -\frac{(-6x-8)+8}{6} = -\frac{-6x-8+8}{6} = -\frac{-6}{6}x = x$ ✓

11. Find the interval notation of the domain of the following function.

$g(w) = \sqrt[4]{1+49w-w^3} = (1+49w-w^3)^{\frac{1}{4}}$

ONE CAN GRAPH $g(w)$ AND LOOK FOR THE DOMAIN, OR USE ALGEBRA.

DOMAIN: "EVEN ROOT" → $\underbrace{1+49w-w^3}_{Y_1} \geq 0$




ZEROS AT: $x \approx -6.99, -0.02, 7.01$

DOMAIN:

$(-\infty, -6.99] \cup [-0.02, 7.01]$

GRAPH ABOVE (>) OR AT (=) THE X-AXIS.

**Extra points**

12. Given $f(x) = \dfrac{x^3 - 7x^2 - 1x}{x - 6}$, determine the equations of any asymptote.

YOU CAN USE A CAS HERE!

$$x-6 \;\overline{)\; x^3 - 7x^2 - x}$$

$x^2 - x - 7$

$x^3 - 6x^2$

$-x^2 - x$

$-x^2 + 6x$

$-7x$

$-7x + 42$

$-42$

$$f(x) = \underbrace{x^2 - x - 7}_{\text{NON-V.A.}} + \frac{-42}{x-6}$$

V.A.

PLUG $x = 6$

$\frac{**}{6}$ ✓

$Y = x^2 - x - 7$

QUADRATIC ASYMPTOTE

$X = 6$

VERT. ASYM.