

Math 221- Spring 2010 - Test 1

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Name: KEY

Instructions. Only calculators are allowed on this examination. Point values of each problem are indicated. Always use the appropriate wording and units of measure in your answers (when applicable). SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

1. (18 pts) Identify any intercept and test for symmetry the function $y = 6 - |x|$. You can not justify your answer with a graph.

X-INTERCEPT, $y=0$: $0 = 6 - |x| \Rightarrow |x| = 6$ $\begin{cases} x=6 \rightarrow (6,0) \\ x=-6 \rightarrow (-6,0) \end{cases}$

Y-INTERCEPT, $x=0$: $y = 6 - 0 = 6 \rightarrow (0,6)$

X-SYMMETRY, PLUG $(x, -y)$: $-y = 6 - |x|$ FAIL (NOT SAME EQ.)

Y-SYMMETRY, PLUG $(-x, y)$: $y = 6 - |-x| \Rightarrow y = 6 - |x|$ ✓

O-SYMMETRY: PLUG $(-x, -y)$: $-y = 6 - |-x| \Rightarrow y = -6 + |x|$
FAIL (NOT SAME EQUATION)

2. (18 pts) Write an equation of the lines through the point $(-6, 4)$, which are respectively parallel and perpendicular to the line $3x + 4y = 7$.

INITIAL LINE: $3x + 4y = 7 \Rightarrow 4y = -3x + 7 \Rightarrow y = -\frac{3}{4}x + \frac{7}{4}$

SLOPE
 $-\frac{3}{4}$

PARALLEL LINE: $y = mx + b \Rightarrow y = -\frac{3}{4}x + b$

PLUG POINT: $4 = -\frac{3}{4}(-6) + b \Rightarrow b = 4 - \frac{9}{2} = -\frac{1}{2}$

SAME
SLOPE

EQUATION: $y = -\frac{3}{4}x - \frac{1}{2}$

PERPENDICULAR: $y - y_1 = m(x - x_1) \Rightarrow y - 4 = \frac{4}{3}(x + 6)$

OPPOSITE
RECIPROCAL
SLOPE

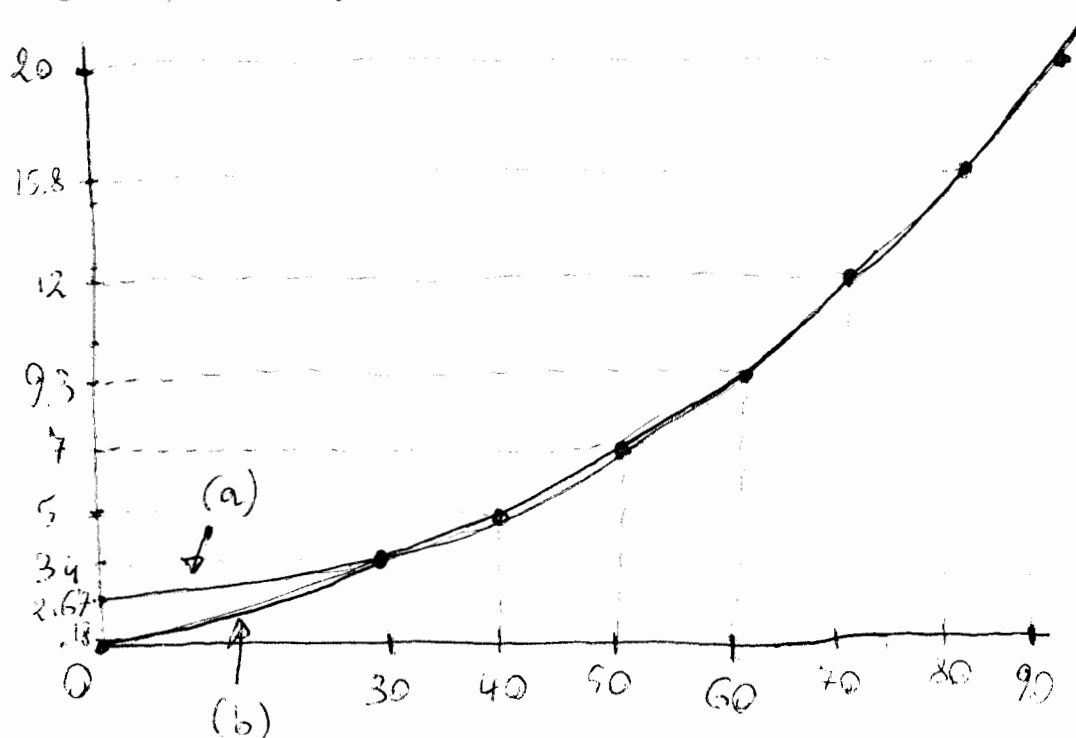
$m = -\frac{1}{-\frac{3}{4}} = \frac{4}{3}$

$y = \frac{4}{3}x + 12$

3. (24 pts) The time t (in seconds) required to attain a speed of s miles per hour from a *standing start* for a Dodge Avenger is shown in the table.

s	30	40	50	60	70	80	90
t	3.4	5	7	9.3	12	15.8	20

- (a) Using your calculator, make a scatter plot of the data (reporting the coordinates of the points along the axis) and find the quadratic model which is the best fit for the data.



- (b) Using the *standing start* point $(0,0)$, repeat part (a) overwriting on the previous graph. Report both models approximating to the fourth decimal place.

(a) $y = .0027x^2 - .0529x + 2.6714$

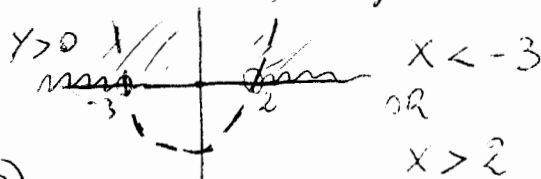
(b) $y = .002x^2 + .0346x + .1827$

4. (18 pts) Find the domain and range, in interval notation, of the function $f(x) = \frac{2\pi}{\sqrt{x^2+x-6}}$.

DOMAIN: $\sqrt{x^2+x-6} \neq 0$ AND $x^2+x-6 \geq 0$, THUS:

$$x^2+x-6 > 0$$

BOUNDARY LINE: $y = x^2+x-6$
 $y = (x-2)(x+3)$

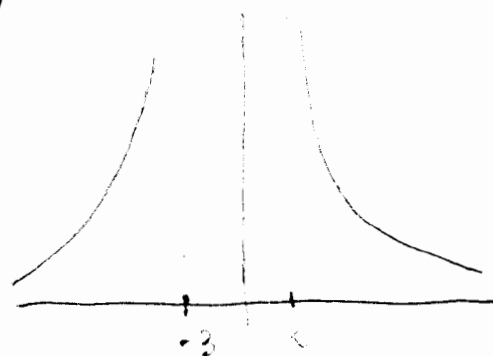


DOMAIN: x IN $(-\infty, -3) \cup (2, \infty)$

RANGE: $\sqrt{\text{---}} > 0$ THEREFORE $f(x) > 0$

RANGE $(0, +\infty)$

GRAPH OF $f(x)$

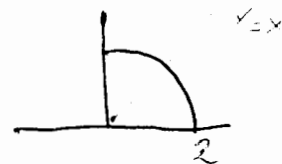
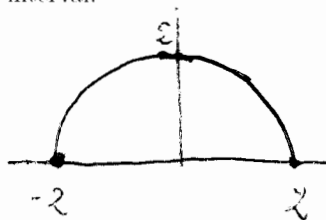


5. (18 pts) Determine a maximal interval in the domain on which $f(x) = \sqrt{4-x^2}$ is one-to-one, then find the inverse function over this interval.

GRAPH OF $f(x)$:

DOMAIN: $[-2, 2]$

RANGE: $[0, 2]$



$f(x)$ IS 1-TO-1 ON $[0, 2]$ AND ON $[-2, 0]$

FROM THE GRAPH, WE SEE THAT $f(x) = \sqrt{4-x^2}$ ON $[0, 2]$ IS THE INVERSE OF ITSELF.

ON $[-2, 0]$: $y = \sqrt{4-x^2} \Rightarrow y^2 = 4-x^2 \Rightarrow x^2 = 4-y^2 \Rightarrow x = \pm\sqrt{4-y^2}$

SINCE $x \leq 0$, THEN $x = \sqrt{x^2} = -\sqrt{4-y^2}$, SWAPPING x, y :

$y = -\sqrt{4-x^2}$ IS THE INVERSE OF $f(x)$.

6. (18 pts) Solve the equation $\arctan(2x-5) = -1$

$$\tan(\arctan(2x-5)) = \tan(-1) \Rightarrow 2x-5 = \tan(-1)$$

$$\Rightarrow x = \frac{5 + \tan(-1)}{2} \approx 1.7213$$

7. (18 pts) Solve the equation $e^{3-4x} = 2$

$$\ln(e^{3-4x}) = \ln 2 \Rightarrow 3-4x = \ln 2 \Rightarrow$$

$$\Rightarrow x = \frac{3 - \ln 2}{4} \approx .5767$$

8. (18 pts) Solve the inequality $\left(\frac{2}{3}\right)^{1-3x} \leq 6$

Since $\log_{\frac{2}{3}}(x)$ is DECREASING, we must switch the direction.

$$\log_{\frac{2}{3}}\left(\left(\frac{2}{3}\right)^{1-3x}\right) \geq \log_{\frac{2}{3}} 6 \Rightarrow 1-3x \geq \log_{\frac{2}{3}} 6 \Rightarrow$$

$$\Rightarrow \frac{-3x}{-3} \geq \frac{-1 + \log_{\frac{2}{3}} 6}{-3} \Rightarrow x \leq \frac{1 - \log_{\frac{2}{3}} 6}{3} \approx 1.8063$$