# Math 221 - Test 4 - Part 1/2 - Fall 2015

Instructor: Dr. Francesco Strazzullo Name KEY

**Instructions.** You **can not** use a graph to justify your answer. Each exercise is worth 10 points.
**SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).**

1. Find the most general antiderivative of the function $f(x) = \dfrac{2x}{x^2+3} + \sin(4x)$, then the particular antiderivative $F(x)$ that satisfies the condition $F(1) = 0$.

$$F(x) = \int \underbrace{\frac{2x}{x^2+3}}_{\substack{u = x^2+3 \\ u' = 2x \to dx = \frac{1}{2x}du}} + \sin(\underbrace{4x}_{u=4x \to dx = \frac{1}{4}du})\,dx = \int \cancel{2x}\frac{1}{u}\frac{du}{\cancel{2x}} + \int \sin u \frac{du}{4} =$$

$$= \ln|u| - \frac{1}{4}\cos u + C = \boxed{\ln(x^2+3) - \frac{1}{4}\cos(4x) + C}$$

PARTICULAR SOLUTION: $0 = F(1) = \ln(1+3) - \frac{1}{4}\cos(4) + C \Rightarrow$

$\Rightarrow C = \frac{1}{4}\cos(4) - \ln 4 \approx -1.55.$

$$F(x) = \ln(x^2+3) - \frac{1}{4}\cos(4x) + \frac{1}{4}\cos 4 - \ln 4.$$

2. Check that the function $y = x^2 - \cos x$ is a solution of the second order ODE
$\underbrace{2y'' + y + \sin x}_{\text{L.H.S.}} = \underbrace{y' + (x-2)^2 + 2x + \cos x.}_{\text{R.H.S.}}$

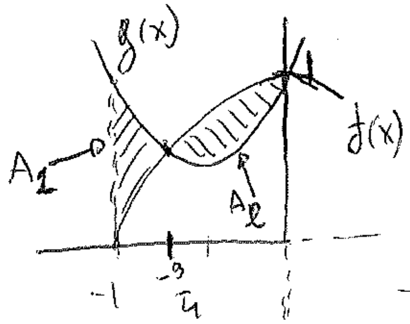
$y' = 2x + \sin x$ ; $y'' = 2 + \cos x$

$$\text{L.H.S} = 2(2+\cos x) + x^2 - \cos x + \sin x = 4 + \cos x + x^2 + \sin x$$

$$\text{R.H.S} = (2x + \sin x) + (x^2 - 4x + 4) + 2x + \cos x$$
$$= \sin x + x^2 + 4 + \cos x = \text{L.H.S} \quad \checkmark$$

NOTE: A TYPO WAS FIXED IN THE ORIGINAL VERSION, WHERE $\text{L.H.S} = 2y'' + y - \sin x$. IN THAT CASE L.H.S. $\neq$ R.H.S AND THE GIVEN FUNCTION WAS NOT A SOLUTION.

3. Compute the area of the region enclosed by the graphs of $f(x) = -3x^2 - 2x + 1$ and $g(x) = x^2 + x + 1$ on the interval $[-1, 0]$.




$$A = A_1 + A_2 = \int_{-1}^{-3/4} g(x) - f(x)\,dx + \int_{-3/4}^{0} f(x) - g(x)\,dx = \int_{-1}^{-3/4} 4x^2 + 3x\,dx + \int_{-3/4}^{0} -4x^2 - 3x\,dx =$$

$$\boxed{f(x) = g(x) \Leftrightarrow 0 = g(x) - f(x) = 4x^2 + 3x \Leftrightarrow x = 0, -\tfrac{3}{4}.}$$

$$= \left[\tfrac{4}{3}x^3 + \tfrac{3}{2}x^2\right]_{-1}^{-3/4} - \left[\tfrac{4}{3}x^3 + \tfrac{3}{2}x^2\right]_{-3/4}^{0}$$

$$= \frac{19}{48} \approx .396$$

4. $\int_1^4 x^2 + \cos(3x - 1)\,dx =$

$$= \left[\frac{x^3}{3}\right]_1^4 + \int_1^4 \cos(3x-1)\,dx = 21 + \int_2^{11} \cos u \frac{du}{3}$$

$$u = 3x - 1 \Rightarrow dx = \tfrac{1}{3}du$$
$$\hookrightarrow \begin{cases} x = 1 \to u = 2 \\ x = 4 \to u = 11 \end{cases}$$

$$= 21 + \tfrac{1}{3}\left[\sin u\right]_2^{11} = 21 + \tfrac{1}{3}\sin 11 - \tfrac{1}{3}\sin 2 \approx 20.364$$

5. A virus is spreading, infecting at a rate modeled by $I'(x) = 500\dfrac{(500-x^2)}{(x^2+5)^2}$ individuals per day, where $I(t)$ is the number of new people infected $t$ days after the first case has been recorded. What is the net-change in new cases recorded between the first and the third day? (Once you setup this problem you can use technology.)

ASKED FOR:

$$I(3) - I(1) = \int_1^3 I'(x)\,dx = \int_1^3 500\,\frac{(500-x^2)}{(x^2+5)^2}\,dx$$

$\overset{GGB}{\approx}$ 6844 (TRUNCATED) NEW CASES.

6. A rocket is lunched from a vertical position while at rest and 100 feet above the sea level. Immediately after lunch the rocket's acceleration is modeled by $a(t) = 2t - 5$, where $t$ is the time in seconds after the rockets is lunched. After 15 seconds the acceleration is modeled by $a(t) = t^2 - 3t - 1$. Twenty seconds after lunch the rocket is only subject to the gravitational force and it starts falling as a free object.

   (a) Express the hight of the rocket as a function of the time (in seconds).

   (b) How long does it take the rocket to reach the 20,000 feet altitude?

(a)

$$a(t) = \begin{cases} 2t-5, & 0 \le t < 15 \\ t^2-3t-1, & 15 \le t < 20 \\ -32, & t \ge 20 \end{cases} \quad ; \; V(0)=0, \; S(0)=100$$

$\boxed{0 \le t < 15}$ $V(t) = \int a(t)\,dt = \int 2t-5\,dt = t^2-5t+C \Rightarrow 0 = C \Rightarrow$

$\Rightarrow V(t) = t^2 - 5t \Rightarrow S(t) = \int V(t)dt = \frac{t^3}{3} - \frac{5}{2}t^2 + C \Rightarrow C = 100$

$\Rightarrow \boxed{S(t) = \frac{1}{3}t^3 - \frac{5}{2}t^2 + 100}$

$\boxed{15 \le t < 20}$ $V(t) = \int t^2-3t-1\,dt = \frac{1}{3}t^3 - \frac{3}{2}t^2 - t + C$, INITIAL CONDITION IS $V(15) = 15^2 - 5\cdot 15 = 150 \Rightarrow 150 = \frac{1}{3}(15)^3 - \frac{3}{2}(15)^2 - 15 + C \Rightarrow C = \frac{-1245}{2}$

$\Rightarrow V(t) = \frac{1}{3}t^3 - \frac{3}{2}t^2 - t - \frac{1245}{2} \Rightarrow S(t) \stackrel{INT}{=} \frac{1}{12}t^4 - \frac{1}{2}t^3 - \frac{1}{2}t^2 - \frac{1245}{2}t + C$

AND $S(15) = \frac{1}{3}(15)^3 - \frac{5}{2}(15)^2 + 100 = \frac{1325}{2} \Rightarrow \frac{1}{12}(15)^4 - \frac{1}{2}(15)^3 - \frac{1}{2}(15)^2 +$

$- \frac{1245}{2}(15) + C = \frac{1325}{2} \Rightarrow C = \frac{30325}{4}$. THEN

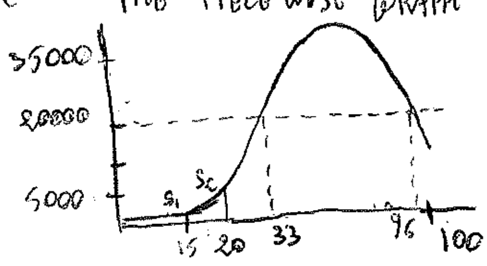
$\boxed{S(t) = \frac{1}{12}t^4 - \frac{1}{2}t^3 - \frac{1}{2}t^2 - \frac{1245}{2}t + \frac{30325}{4}}$

$\boxed{t \ge 20}$ $V(t) \stackrel{INT}{=} -32t + C$ AND $V(20) = \frac{8545}{6} \Rightarrow C = \frac{12385}{6} \Rightarrow$

$\Rightarrow V(t) = -32t + \frac{12385}{6} \Rightarrow S(t) = -16t^2 + \frac{12385}{6}t + C$ AND

$S(20) = \frac{51175}{12} \Rightarrow C = -\frac{122475}{4} \Rightarrow \boxed{S(t) = -16t^2 + \frac{12385}{6}t - \frac{122475}{4}}$

(b) THE PIECEWISE GRAPH




20,000 FT ARE REACHED DURING THE LAST PHASE:

$-16t^2 + \frac{12385}{6}t - \frac{122475}{4} = 20000 \Rightarrow \boxed{t \approx 33,96 \text{ SECONDS}}$

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7. $\int x\sqrt{x+1}\,dx =$ $\int (u-1)\sqrt{u}\,du = \int u^{\frac{3}{2}} - u^{\frac{1}{2}}\,du = \frac{u^{5/2}}{5/2} - \frac{u^{3/2}}{3/2} + C$

$u = x+1 \Rightarrow du = dx$
$\hookrightarrow x = u-1$

$= \frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2} + C$

8. $\int_{\frac{\pi}{2}}^{\pi} x\cos(x^2+1)\,dx =$ $\int_{\frac{\pi^2}{4}+1}^{\pi^2+1} \cancel{x}\cos u \cdot \frac{1}{2\cancel{x}}\,du = \frac{1}{2}\Big[\sin u\Big]_{\frac{\pi^2}{4}+1}^{\pi^2+1}$

$u = x^2+1 \Rightarrow u' = 2x \Rightarrow dx = \frac{1}{2x}du$

$\hookrightarrow x = \frac{\pi}{2} \Rightarrow u = \frac{\pi^2}{4}+1$

new end points $\hookrightarrow x = \pi \Rightarrow u = \pi^2+1$

$= \frac{1}{2}\left(\sin(\pi^2+1) - \sin\left(\frac{\pi^2}{4}+1\right)\right) \approx -.34$