Math 102-040 - Fall 2009 - Test 2

Instructor: Dr. Francesco Strazzullo Name KEY

Instructions. Only calculators are allowed on this examination. Point values of each problem are indicated. **Always use the appropriate wording and units of measure in your answers (when applicable). SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).**

1. Find the x-coordinate of the vertex of the parabola $y = -0.2x^2 - 32x + 2$.

$$\text{VERTEX} = (h, k) \Rightarrow h = \frac{-b}{2a} = \frac{-(-32)}{2 \cdot .2} = -\frac{32}{.4} = -\frac{32}{4/10}$$

$$= -\frac{32}{4} \cdot 10 = -80$$

WE COULD USE A GRAPHING CALCULATOR.

2. The profit for a product can be described by the function $P(x) = 40x - 3000 - .01x^2$ (measured in dollars), where x is the number of units produced and sold. To maximize profit, how many units must be produced and sold? What is the maximum possible profit?

P(X) IS A DOWNWARD PARABOLA, BECAUSE THE NUMERICAL COEFFICIENT OF X^2 IS NEGATIVE, THEN THE MAXIMUM IS AT THE VERTEX.

GRAPHING CALCULATOR OR COMPLETING THE SQUARE OR ALGEBRA.

$$\text{VERTEX} = (h, k) \Rightarrow h = \frac{-b}{2a} = \frac{-40}{2 \cdot (-.01)} = \frac{20}{1/100} = 2000$$

$$k = P(h) = P(2000) = 40\ 2000 - 3000 - .01 \cdot 2000^2 = 37{,}000$$

THE MAXIMUM PROFIT POSSIBLE IS OF 37,000 DOLLARS AND IT IS ACHIEVED WHEN SELLING 2000 UNITS.

3. Solve the equation $2x^2 + 2x - 12 = 0$. (Show your work)

CALCULATOR (WITH GRAPH) OR ALGEBRA:

$$\frac{2x^2+2x-12}{2} = \frac{0}{2} \rightarrow x^2 \underbrace{+}_{\text{SUM}} x \underbrace{-6}_{\text{PRODUCT}} = 0 \rightarrow$$

PRODUCT $-3 \cdot (-2)$

$$\rightarrow (x-2)(x+3) = 0 \begin{cases} x-2=0 \rightarrow x=2 \\ x+3=0 \rightarrow x=-3 \end{cases}$$

4. The profit for a product is given by $P(x) = -12x^2 + 1320x - 21{,}600$ (measured in dollars). where x is the number of units produced and sold. How many units give break even for this product?

BREAK EVEN IS $P(x)=0$.

$$\frac{-12x^2+1320x-21{,}600=0}{-12} \rightarrow x^2 \underbrace{-110}_{\text{SUM}} x \underbrace{+1800}_{\text{PRODUCT}} = 0$$

PRODUCT $-90, -20$

$$(x-90)(x-20)=0 \begin{cases} x-90=0 \rightarrow x=90 \\ x-20=0 \rightarrow x=20 \end{cases}$$

WE GET BREAK EVEN WHEN PRODUCING AND SELLING 90 OR 20 UNITS.

5. The 2004 U.S. federal income tax owed by a married couple filing jointly can be found from the following table, where the percentage is taken on the taxable income.

If Taxable income is between	Taxable due is
\$0 – \$15,650	\$0.00 + 10%
\$15,650 – \$63,700	\$1,565 + 15%
\$63,700 – \$128,500	\$8,772.50 + 25%
\$128,500 – \$195,850	\$24,872.5 + 28%

(a) Write the piecewise-defined function T with input x that models the federal tax dollars owed as a function of x, the taxable income dollars earned, with $0 < x \leq 128,500$.

$$T(x) = \begin{cases} .1X, & 0 < X \leq 15,650 \\ 1565 + .15X, & 15,650 < X \leq 63,700 \\ 8772.5 + .25X, & 63700 < X \leq 128,500 \end{cases}$$

(b) Use the function to find $T(42,000)$.

$$T(42,000) = 1565 + .15 \cdot 42000 = 7865$$

(c) Find the tax owed on a taxable income of \$68,000.

IT IS ASKED FOR $T(68,000) = 8772.5 + .25 \cdot 68000 = 25772.5$

THE TAX OWED ON A TAXABLE INCOME OF \$ 68,000 IS 25,772.5 DOLLARS

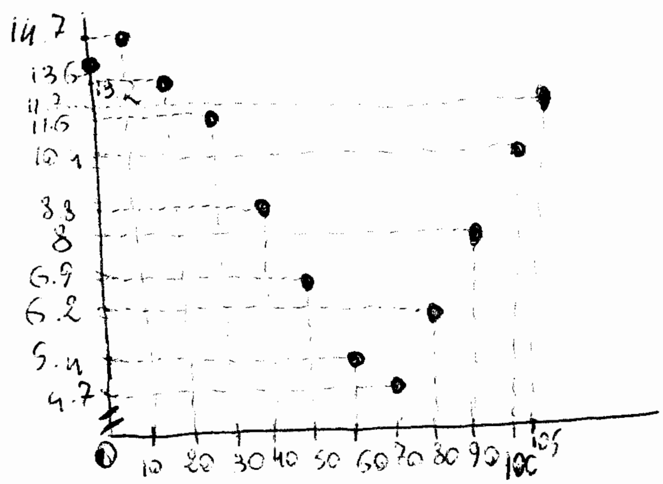
6. The following table gives the percent of U.S. population that is foreign born, for selected years from 1900 to 2005.

Year	1900	1910	1920	1930	1940	1950
Foreign born (%)	13.6	14.7	13.2	11.6	8.8	6.9

Year	1960	1970	1980	1990	2000	2005
Foreign born (%)	5.4	4.7	6.2	8.0	10.4	11.7

(a) Make a scatter plot of the data, with x equal to the number of years past 1900 and y equal to the percent. (Clearly report the coordinates of the points)

WE MUST SET X = "# YEARS PAST 1900". FOR EXAMPLE X = 105 IN 200



(b) Using your calculator, find the quadratic model which is the best fit for the data. Report your answer to 4 decimal places.

SAY $Y = ax^2 + bx + c$ IS OUR MODEL. $a = .002384$.
IF THE WRONG CHOICE "X = YEAR" IS MADE, THEN b AND C ARE OFF.
$b = -.306578$; $C = 16.491969$. THUS
$Y = .0024x^2 - .3066x + 16.492$

(c) Use the function to estimate the percent for 2010.

2010 IS $X = 110$

$Y = P(110) = .0024 \cdot 110^2 - .3066 \cdot 100 + 16.492 = 11.806$

NOTE THAT THE UNROUNDED ANSWER WOULD BE 11.624 %

THE ESTIMATED PERCENT OF FOREIGN BORN US POPULATION IS 11.806.

7. Let $f(x) = \sqrt{x-1}$ and $g(x) = 2x - 7$. Compute $(f \circ g)(x)$ and $g(f(5))$.

$$(f \circ g)(x) = f(g(x)) = \sqrt{(2x-7)-1} = \sqrt{2x-8}$$

$$f(5) = \sqrt{5-1} = \sqrt{4} = 2 \quad \text{THUS,}$$

$$g(f(5)) = g(2) = 2 \cdot 2 - 7 = -3$$

8. Suppose the total weekly cost for the production and sale of x bicycles is $C(x) = 23x^2 + 3420$ dollars and that the total revenue is given by $R(x) = 89x$ dollars, where x is the number of bicycles. Write the equation of the function that models the weekly profit from the production and sale of x bicycles, then compute the profit for producing and selling 150 bicycles.

$$P = R - C = 89x - (23x^2 + 3420) = 89x - 23x^2 - 3420$$

$$\rightarrow \boxed{P(x) = -23x^2 + 89x - 3420}$$

$$P(150) = -507,570$$

WHEN SELLING 150 BICYCLES THE COST IS BIGGER THAN THE REVENUE, THAT IS, THERE IS A NEGATIVE PROFIT OF 507,570 DOLLARS.

9. Find the inverse of the function $g(x) = 4x + 1$. → $Y = 4X + 1$

1) SWAP X AND Y: $X = 4Y + 1$
$\quad -1 \qquad -1$

2) SOLVE FOR Y:
$$\frac{X-1}{4} = \frac{4Y}{4} \rightarrow Y = \frac{X-1}{4}$$

3) THE INVERSE OF $g(x)$ IS $f(x) = g^{-1}(x) = \frac{X-1}{4}$

(CHECK THAT $f(g(x)) = X$ AND $g(f(x)) = X$)

10. The intensity of illumination of a light is a function of the distance from the light. For a given light, the intensity is given by $I(x) = \frac{300,000}{x^2}$ candle-power, where x is the distance in feet from the light. Express the distance as a function of the intensity and compute the distance from the lamp when the intensity of the light is 75,000 candle-power.

$Y = \frac{300,000}{X^2}$, Y IS INTENSITY AND X DISTANCE

I) BY THE CONTEXT $X > 0$ SO THAT $I(X)$ IS ONE-TO-ONE.

TO WRITE THE DISTANCE AS A FUNCTION OF THE INTENSITY WE MUST COMPUTE THE INVERSE OF $I(X)$.

1) SWAP X AND Y: $X = \frac{300,000}{Y^2}$

SO THAT NOW X IS INTENSITY AND Y THE DISTANCE.

2) SOLVE FOR Y: $Y^2 \frac{X}{X} = \frac{300,000}{\cancel{Y^2}} \frac{\cancel{Y^2}}{X} \rightarrow Y^2 = \frac{300,000}{X}$

Y IS DISTANCE, SO $Y > 0$ AND WE TAKE THE POSITIVE ROOT

$Y = \sqrt{\frac{300,000}{X}}$. SO $D(X) = \sqrt{\frac{300,000}{X}}$ IS THE ANSWER.

II) $\Delta \rightarrow$ $D(75,000) = \sqrt{\frac{300,000}{75,000}} = 2$. SO AT 2 ft FROM THE LAMP THE INTENSITY IS 75,000 CANDLE-POWER.