MAT 221 - Fall 2017 - Exam 2 – Take Home Part

Instructor: Dr. Francesco Strazzullo **Name** ___KSY___

I did not receive third party help in completing this test. *Signature*__________

Instructions. You are expected to use a graphing calculator or software to complete some problems. Files can be downloaded and uploaded to the Eagleweb Coursework page for this assignment. Upload files or sketch any graph that you use or tables of input/output, **approximating up to the fourth decimal place**. Each problem is worth 10 points, unless otherwise specified.

SHOW YOUR WORK NEATLY, PLEASE. (no work = no points)

1. Find the value of the limit

$$\lim_{x\to 0}\frac{\sin(3x)}{6x} = \lim_{x\to 0}\frac{1}{2}\frac{\sin(3x)}{3x} = \frac{1}{2}\lim_{z\to 0}\frac{\sin z}{z} = \frac{1}{2}\cdot 1 = \frac{1}{2}$$

$\lim_{x\to 0} 3x = 3\cdot 0 = 0$ $\quad z = 3x$

KNOWN LIMIT

2. Find the value of the limit

$$\lim_{x\to -4}\left(\frac{1}{x+4}-\frac{2x}{x^2-4}\right) = \lim_{x\to -4}\frac{x^2-4\overbrace{-2x(x+4)}^{-2x^2-8x}}{(x^2-4)(x+4)} = \lim_{x\to -4}\frac{-x^2-8x-4}{(x^2-4)(x+4)}$$

$x^2-4=(x-2)(x+2)$

LCD $=(x^2-4)\cdot(x+4)=$

$$\lim_{x\to -4^-}\frac{-(x^2+8x+4)}{(x^2-4)(x+4)} \quad \frac{*}{\ominus} = -\infty \quad \frac{-}{+\,-}$$

TEST $x=-4.1$

$$\lim_{x\to -4^+}\frac{-(x^2+8x+4)}{(x+4)(x^2-4)} \quad \frac{*}{\oplus} = +\infty \quad \frac{-}{+\,+}$$

TEST $x=-3.9$

$\rightarrow \lim_{x\to -4}\left(\frac{1}{x+4}-\frac{2x}{x^2-4}\right)$ IS UNDEFINED

3. Find the linear approximation to $f(x) = \ln(3x - 2x^2)$ at $x = 1$.

$f'(x) = \dfrac{3-4x}{3x-2x^2} \Rightarrow f'(1) = -1$; $f(1) = 0$

$L(x) = f(1) + f'(1)\cdot(x-1) = -(x-1) = -x+1$

4. Find $\dfrac{dy}{dx}$ when $y = x^{\ln x}$. $\longrightarrow$ $\ln y = \ln(x^{\ln x}) = \ln x \cdot \ln x \Rightarrow$

$\Rightarrow \dfrac{d}{dx}[\ln y] = \dfrac{d}{dx}[(\ln x)^2] \Rightarrow \dfrac{y'}{y} = 2\cdot(\ln x)^{2-1}\cdot\dfrac{1}{x} \Rightarrow$

$\Rightarrow \dfrac{dy}{dx} = y' = \dfrac{2\ln x}{x}\cdot \cancel{y} \Rightarrow y' = 2(\ln x)\cdot x^{-1}\cdot x^{\ln x} \Rightarrow$

$\Rightarrow y' = 2x^{\ln(x)-1}\cdot\ln(x)$

5. Find the tangent line to the parametric curve $x = 2t - t^3$, $y = t^2 - 3t$ at the point $(-4, -2)$. Use symbolic notation, do not approximate.

$\begin{cases} x = f(t) \\ y = g(t) \end{cases} \Rightarrow \dfrac{dy}{dx} = \dfrac{g'(t)}{f'(t)}$ HERE $\begin{cases} f(t) = 2t - t^3 = -4 \\ g(t) = t^2 - 3t = -2 \end{cases}$ POINT

$\begin{cases} 2t - t^3 = -4 \\ t^2 - 3t + 2 = 0 \Rightarrow (t-1)(t-2) = 0 \Rightarrow t = 1, 2 \end{cases}$ TEST $\begin{cases} 2(1) - 1 \neq -4 \\ 2(2) - 2^3 = -4 \checkmark \end{cases}$

$\left.\begin{array}{l} f'(t) = 2 - 3t^2 \Rightarrow f'(2) = -10 \\ g'(t) = 2t - 3 \Rightarrow g'(2) = 1 \end{array}\right] \Rightarrow \left.\dfrac{dy}{dx}\right|_{(-4,-2)} = \dfrac{g'(2)}{f'(2)} = -\dfrac{1}{10} = y_0'$

$L(x) = y_0 + y_0' \cdot (x - x_0) = -2 - \frac{1}{10}(x + 4) = -\frac{1}{10}x - \frac{12}{5}$

6. Let $f(x) = \begin{cases} 3x^2 - 2, & \text{if } x < -1 \\ 2x + 1, & \text{if } -1 \le x \le 2 \\ x^3 + 1, & \text{if } x > 2 \end{cases}$. Complete the following parts.

(a) $f(-1) = 2(-1) + 1 = -1$

(b) $\lim_{x \to -1} f(x) =$

(c) Is $f(x)$ continuous at -1? If not, can we remove this discontinuity?

(b) $\left.\begin{array}{l} \lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} 3x^2 - 2 = 3 - 2 = 1 \\ \lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} 2x + 1 = -1 \end{array}\right] \Rightarrow \lim_{x \to -1} f(x)$ IS UNDEFINED

(c) $f(x)$ IS NOT CONTINUOUS AT $x = -1$, AND THE DISCONTINUITY IS NOT REMOVABLE BECAUSE THE LIMIT IS UNDEFINED AT $x = -1$.

7. The table below gives the violent crime rate (per 100,000 people) for a particular state every five years from 1970 to 2010.

Year	1970	1975	1980	1985	1990	1995	2000	2005	2010
X	10	15	20	25	30	35	40	45	50
Violent Crime Rate Y	4.9	5.1	6	7.4	9	10.5	11.7	12.4	12.2

Consider x to be the number of years after 1960, and y to be the violent crime rate during that year.

(a) Use average rates of change to estimate the rate of change of the yearly violent crime rate in 1990 (rounded to the first decimal place).

$1990 \to x = 30 \Rightarrow f'(30) \approx \frac{f(35)-f(25)}{2\cdot(5)} = \frac{10.5-7.4}{10} = 0.31 \approx .3$

$Y = f(x)$

INCREASE .3 VIOLENT CRIMES PER 100,000 PEOPLE PER YEAR

(b) Use technology to compute the 3rd degree polynomial regression and the trigonometric (sine) regression that best fit the data above. Report the correlation coefficients (round to 4 decimal places) and state which regression line is the best model in this context.

CUBIC: $Y = -0.0003x^3 + .0298x^2 - .5418x + 7.6802$, $R^2 = .99997$

SINE: $y = 8.6291 + 3.7468\sin(.0876x - 2.5301)$, $R^2 = .99988$

THESE MODELS ARE EQUALLY WELL FIT FOR THESE DATA. IN THIS CONTEXT, THE SINE MODEL MAKES MORE SENSE BECAUSE THE CUBIC ONE SHOWS VERY STEEP DECLINES RIGHT OUTSIDE THE BOUNDARIES ($t < 10$ OR $t > 50$). NOTE WHAT HAPPENS TO THE APPROXIMATED MODELS.

(c) Use the best model from part (b) to estimate the rate of change of the yearly violent crime rate in 1990 and compare it to the one you found it in part (a).

$f'(x) = 3.7468(.0876)\cdot\cos(.0876x - 2.5301)$

$f'(30) = .3266$ VIOLENT CRIMES PER 100,000 PEOPLE PER YEAR.

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8. Express $\frac{dy}{dx}$ when $3y^2x - x = x^3$.

HONOR: WRITE THE EQUATION OF THE NORMAL LINE AT $(1, \sqrt{2/3})$

NOTE: THIS IMPLICIT FUNCTION IS MADE UP OF TWO DISTINCT EQUATIONS: $x(3y^2-1) = x^3 \iff \left[\begin{array}{l} x=0 \\ \text{OR} \\ 3y^2-1=x^2 \end{array}\right.$

$x=0$ IS A VERTICAL LINE AND $\frac{dy}{dx}$ IS UNDEFINED.

$$\frac{d}{dx}[3y^2-1] = \frac{d}{dx}[x^2] \Rightarrow 3(2y)\cdot y' = 2x \Rightarrow$$

$$\Rightarrow y' = \frac{x}{3y}$$

HONOR $\left.\frac{dy}{dx}\right|_{(1,\sqrt{2/3})} = \frac{1}{3\sqrt{\frac{2}{3}}} \Rightarrow$ SLOPE OF NORMAL $= -3\sqrt{\frac{2}{3}} =$

$$= -3\frac{\sqrt{6}}{3} = -\sqrt{6}.$$

EQ- NORMAL: $y = \sqrt{\frac{2}{3}} - \sqrt{6}(x-1)$

$$= -\sqrt{6}x + \sqrt{6} + \frac{\sqrt{6}}{3}$$

THEN $y = -\sqrt{6}x + \frac{4}{3}\sqrt{6}$

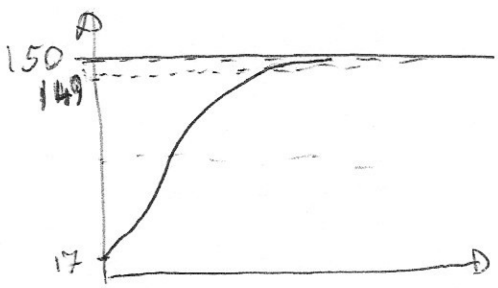
9. Let $P(t) = \frac{150}{1+7.5e^{-0.3t}}$ be the population of domestic cats in Waleska t years after 1883. What is the limiting size L (or *carrying capacity*) of this population? Approximately (to the nearest integer), in which year can you say that the population has reached its carrying capacity, that is $P = L - 1$?

$$L = \lim_{t \to \infty} P(t) = \frac{150}{1 + 7.5 \lim_{t\to\infty} e^{-0.3t}}$$

$$\lim_{t\to\infty} (-0.3t) = -0.3 \cdot \lim_{t\to\infty} t = -0.3 \cdot \infty = -\infty \Rightarrow$$

$$\Rightarrow \lim_{t\to\infty} e^{-0.3t} = \lim_{z \to -\infty} e^{z} = 0$$

$$\Rightarrow L = \frac{150}{1 + 7.5 \cdot 0} = 150 \text{ CATS}$$



POPULATION GROWS TO LIMITING SIZE L

SOLVE $P(t) = 150 - 1 = 149$ GRAPHICALLY $\Rightarrow t \approx 23.39$

IN 1906, AFTER ABOUT 23 YEARS, THERE WILL BE 149 CATS.