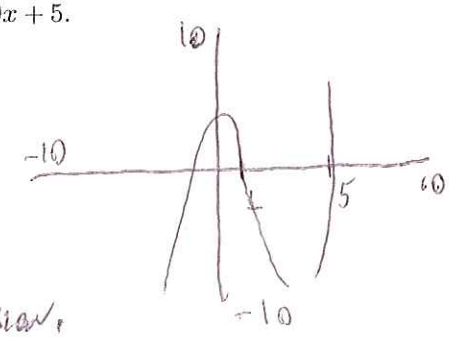
Math 102 - Spring 2012 - Test 3

Instructor: Dr. Francesco Strazzullo   My Name KEY

I certify that I did not receive third party help in completing this test. (*sign*)__________

Instructions. If you use graphic methods, sketch the graphs and label significant points, like intersection points or intercepts. Each exercise is worth 10 points, unless otherwise specified. This is a take home test which covers the 3/26/2012 contact time. *Always use the appropriate wording and units of measure in your answers (when applicable).*
SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

1. Find the rational root(s) of the polynomial $3x^3 - 17x^2 + 9x + 5$.



FROM GRAPH OR TABLE WE CAN FIND
THE INTEGRAL ROOTS: $X = 1$, $X = 5$

OR AMONG THE DIVISORS OF $3 \cdot 5 = 15$

NOW WE CAN USE LONG (OR SYNTHETIC) DIVISION,

BOTH $(X-1)$ AND $(X-5)$ DIVIDE $P(X) = 3X^3 - 17X^2 + 9X + 5$

THEREFORE $(X-1)(X-5) = X^2 - 5X - X + 5 = X^2 - 6X + 5$ DIVIDES $P(X)$.

LONG DIVISION:

$3X^3/X^2 \quad X^2/X^2$

$$\begin{array}{r|l} & 3X + 1 \\ \hline X^2 - 6X + 5 & 3X^3 - 17X^2 + 9X + 5 \\ & 3X^3 - 18X^2 + 15X \\ \hline & \quad X^2 - 6X + 5 \\ & \quad X^2 - 6X + 5 \\ \hline & \qquad 0 \end{array}$$

✓

THUS

$P(X) = (X^2 - 6X + 5)(3X + 1)$

$= \underbrace{(X-1)(X-5)}_{\text{WE ALREADY KNOW THE ROOTS } X = 1, 5}(3X+1)$

$3X + 1 = 0$

$\downarrow$

$3X = -1$

$\downarrow$

$X = -\frac{1}{3}$

$$\boxed{\text{RATIONAL ROOTS OF } P(X): \quad X = 1,\ 5,\ -\tfrac{1}{3}}$$

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2. Completely factor the polynomial $P(x) = 6x^4 + 27x^3 - 27x^2 - 54x + 30$. You can use the fact that $3x^2 - 6$ divides $P(x)$.

LONG DIVISION: $6x^4/3x^2$ $\quad 27x^3/3x^2$ $\quad -15x^2/3x^2$

$$
\begin{array}{r|l}
 & 2x^2 + 9x - 5 \\
\hline
3x^2-6 & 6x^4 + 27x^3 - 27x^2 - 54x + 30 \\
 & 6x^4 \qquad\qquad - 12x^2 \\
\hline
 & \qquad 27x^3 - 15x^2 - 54x + 30 \\
 & \qquad 27x^3 \qquad\quad - 54x \\
\hline
 & \qquad\qquad -15x^2 \qquad + 30 \\
 & \qquad\qquad -15x^2 \qquad + 30 \\
\hline
 & \qquad\qquad\qquad 0 \ \checkmark
\end{array}
$$

THUS:

$P(x) = (3x^2-6)(2x^2+9x-5)$

WE NEED TO FACTOR THESE TWO QUADRATIC POLYNOMIALS

$3x^2 - 6 = 3\underbrace{(x^2-2)}_{\text{DIFFERENCE OF SQUARES}} = 3(x^2 - (\sqrt{2})^2)$

$= 3(x-\sqrt{2})(x+\sqrt{2})$.

FACTOR

$\underbrace{2x^2+9x-5}$, PRODUCT $= 2(-5) = -10$, SUM $= 9$ $\rightarrow 10, -1$

$\Vert$

$2x^2 + 10x - x - 5 = 2x(x+5) - (x+5) = (2x-1)(x+5)$

$$\boxed{P(x) = 3(x-\sqrt{2})(x+\sqrt{2})(2x-1)(x+5)}$$

3. Perform the long division $\dfrac{x^5 - 16x^3 + 4x + 8}{x^3 - 1}$.

x^5/x^3 $\quad -16x^3/x^3$

$$
\begin{array}{r|l}
 & x^2 - 16 \\
\hline
x^3-1 & x^5 - 16x^3 + 4x + 8 \\
 & x^5 \qquad\quad - x^2 \\
\hline
 & \quad -16x^3 + x^2 + 4x + 8 \\
 & \quad -16x^3 \qquad\qquad + 16 \\
\hline
 & \qquad\quad x^2 + 4x - 8
\end{array}
$$

$$\boxed{\frac{x^5 - 16x^3 + 4x + 8}{x^3 - 1} = x^2 - 16 + \frac{x^2 + 4x - 8}{x^3 - 1}}$$

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4. The cost in thousand dollars for producing x hundred cars at a local factory can be modeled by the function $y = x^4 - 5x^3 + 2x + 70$.

   (a) What is the cost for producing 500 cars?

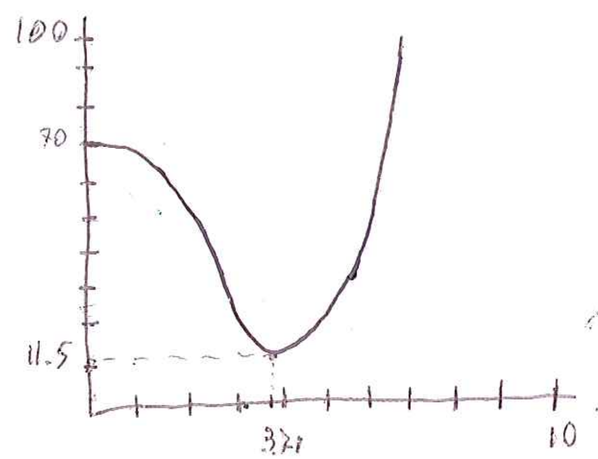
   (b) Find out the level of production that minimizes the cost and such a minimum cost.

X = " HUNDRED CARS " → a) X = 5 , Y(5) = 80 →

→ PRODUCING 500 CARS WILL COST $ 80,000.

b) USE THE GRAPH , THEN

$\boxed{2^{ND}}$ + $\boxed{\text{TRACE}}$ + $\boxed{3}$



TO FIND X = 3.713, Y = 11.546

WE NEED TO ACTUALIZE THIS , USING THE FACT THAT X IS HUNDRED CARS :

X = 3.71 → Y = 11.546

X = 3.72 → Y = 11.547

THEREFORE THE ACTUAL MINIMUM IS (3.71 , 11.546) , THAT IS A MINIMUM COST OF $ 11,546 WHEN 371 CARS ARE PRODUCED.

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5. The effectiveness (on a 0 to 10 scale) of a medication x hours after its administration is given by

$$E(x) = \frac{113 + 201x - 12x^2}{3x^2 + 6x + 28}.$$

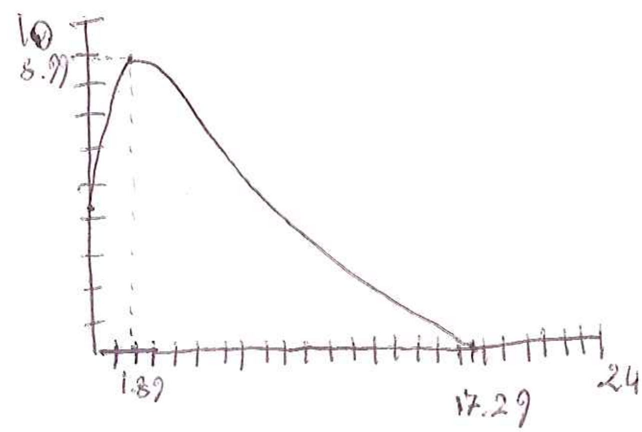
(a) How long does it take the medication to be most effective?

(b) How long after its administration is the medication ineffective?

a) ASKING FOR THE MAXIMUM OF $E(x) = (113+201X-12X^2)/(3X^2+6X+28)$

b) ASKING FOR THE X-INTERCEPT, THAT IS X FOR WHICH $E(X)=0$.

USE THE GRAPH



a) $\boxed{2^{ND}} + \boxed{\text{TRACE}} + \boxed{4}$

$X = 1.89$, $Y = 8.99$

ACTUALIZE:

$X = 1 \to Y = 8.16$

$X = 2 \to Y = 8.98$

AFTER ABOUT 2 HOURS THE MEDICATION IS MOST EFFECTIVE.

b) $\boxed{2^{ND}} + \boxed{\text{TRACE}} + \boxed{2}$ GIVES $X = 17.29$, THEREFORE

AFTER ABOUT 17 HOURS THE MEDICATION HAS LOST ITS EFFECTIVENESS.

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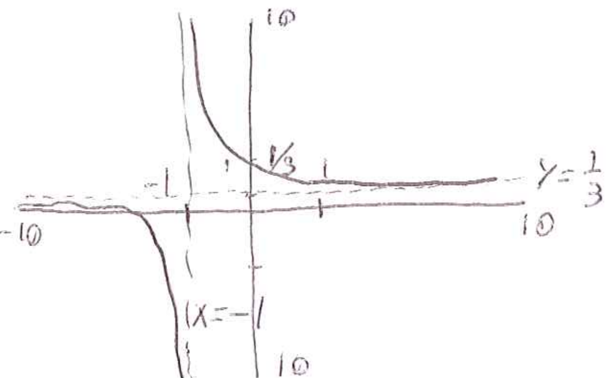
6. For the following rational functions, use algebra to find the domain, then use algebra or technology to find (if any) the asymptotes (vertical, horizontal, or slant). (Each part is worth 10 points)

(a) $f(x) = \dfrac{x^2+2x-3}{3x^2-3} = \dfrac{(x+3)(x-1)}{3(x+1)(x-1)}$



DOMAIN: $3x^2-3 \neq 0$

$3x^2-3=0 \to 3(x^2-1)=0$

$\to x^2-1=0 \to x^2=1 \to x=\pm 1$

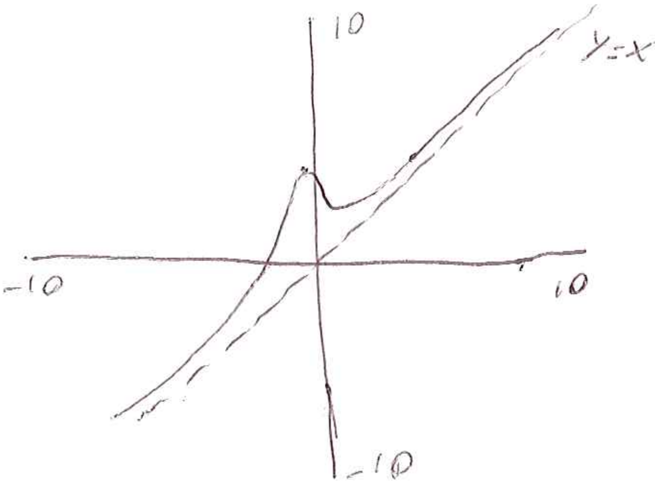
DOMAIN: $x \neq \pm 1$

V.A. I) PLUG $x=1 \to \frac{1+2-3}{3-3} = \frac{0}{0}$ → MIGHT NOT BE A V.A., THEN FROM GRAPH AND FACTORIZATION WE SEE THAT THIS IS NOT A V.A.

II) PLUG $x=-1 \to \frac{1-2-3}{3-3} = \frac{-4}{0}$ → V.A.

"DEGREE NUMERATOR" = "DEG. DEN." → H.A. $y = \frac{1x^2}{3x^2} = \frac{1}{3}$.

(b) $f(x) = \dfrac{x^3-x+5}{x^2+1}$



DOMAIN: $x^2+1 \neq 0$

SOLVE $x^2+1=0 \to x^2=-1 \to x=\pm i$

NOT REAL → DOMAIN: ALL REAL NUMBERS

→ NONE V.A.

"DEGREE NUM" = "DEG. DEN" + 1 → SLANT ASYMPTOTE.

USE LONG DIVISION TO FIND SLANT ASYMPTOTES

$$\begin{array}{r|l} & x \\ \hline x^2+1 & x^3-x+5 \\ & x^3+x \\ \hline & -2x+5 \end{array}$$

REMAINDER

x → $y=x$ IS THE SLANT ASYMPTOTE

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7. Let $f(x) = 4x - x^3$ and $g(x) = 3x + 7$. Compute $(g - f)(x)$ and $(f \cdot g)(-1)$.

$(g-f)(x) = g(x) - f(x) = (3x+7) - (4x - x^3) = 3x + 7 - 4x + x^3$

$= x^3 - x + 7$

$(f \cdot g)(-1) = f(-1) \cdot g(-1) = (4(-1) - (-1)^3)(3(-1)+7) = -3(4) = -12$

8. Let $g(x) = 5x - 1$ and $f(x) = x^2 + 3$. Compute $(f \circ g)(x)$ and $g(f(-1))$.

$(f \circ g)(x) = f(g(x)) = (5x-1)^2 + 3 = (5x)^2 + 2(-1)(5x) + (-1)^2 + 3$

$= 25x^2 - 10x + 4$

$g(f(-1)) = 5((-1)^2 + 3) - 1 = 5(4) - 1 = 19$

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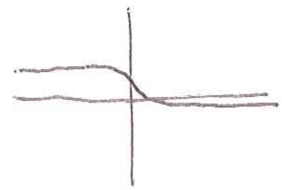
9. Find the inverse of the function $f(x) = 1 - \sqrt[5]{2x}$ and check your result.

$f(x)$ IS 1-TO-1 BECAUSE IT PASSES THE HORIZONTAL LINE TEST.



1) SOLVE FOR X: $Y = 1 - \sqrt[5]{2X}$

$$Y - 1 = -\sqrt[5]{2X} \rightarrow (-Y+1)^5 = \left(\sqrt[5]{2X}\right)^5$$

$$\rightarrow (1-Y)^5 = 2X \rightarrow X = \frac{(1-Y)^5}{2}$$

2) SWAP X AND Y: $Y = \frac{(1-X)^5}{2}$

THE INVERSE FUNCTION IS $f^{-1}(X) = \frac{(1-X)^5}{2}$

CHECK: I) $f(f^{-1}(X)) = 1 - \sqrt[5]{\cancel{2}\left(\frac{(1-X)^5}{\cancel{2}}\right)}$

$$= 1 - \cancel{\sqrt[5]{(1-X)^5}} = 1 - (1-X) = 1 - 1 + X$$

$$= X \quad \checkmark$$

II) $f^{-1}(f(X)) = \frac{\left(1 - \left(1 - \sqrt[5]{2X}\right)\right)^5}{2} = \frac{\left(1 - 1 + \sqrt[5]{2X}\right)^5}{2}$

$$= \frac{\left(\cancel{\sqrt[5]{2X}}\right)^{\cancel{5}}}{2} = \frac{2X}{2} = X \quad \checkmark$$

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