Mat321 – Spring 2019 –Exam1-Home

Instructor: Dr. Francesco Strazzullo **Name**________________________________

I certify that I did not receive third party help in *completing* this test (sign)____________________________

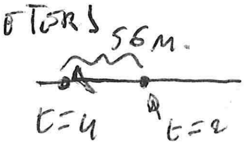
Instructions. Technology and instructor's notes (including the formula sheets from our book) are allowed on this exam. Each problem is worth 10 points. If you use notes or formula sheets, make a reference. When using technology describe which commands (or keys typed) you used or print out your worksheet.

SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

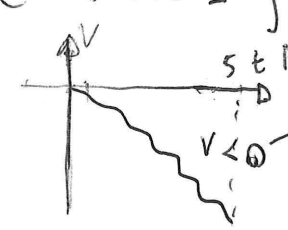
1. A particle travels along a line. Its velocity in meters per second is given by $v(t) = \sin(4\pi t) - 3t^2$.
   (a) Find the displacement during the [2, 4] time frame; and
   (b) the distance traveled by the particle from $t = 1$ to $t = 5$.

(a) DISPL. $= s(4) - s(2) = \int_2^4 s'(t)dt = \int_2^4 \sin(4\pi t) - 3t^2 = \left[-\frac{1}{4\pi}\cos(4\pi t) - t^3\right]_2^4$

$= \cancel{-\frac{1}{4\pi}\cos(16\pi)} - 4^3 + \cancel{\frac{1}{4\pi}\cos(8\pi)} + 2^3 = -56$ METERS



(b) DIST. $= \int_1^5 |v(t)|\,dt = \int_1^5 -v(t)\,dt = \int_1^5 -\sin(4\pi t) + 3t^2 =$



$= \left[\frac{1}{4\pi}\cos(4\pi t) + t^3\right]_1^5 = \frac{\cos(20\pi)}{4\pi} + 5^3 - \cos(4\pi) - 1^3 =$

$= 5^3 - 1^3 = 124$ METERS

2. The marginal profit for renting x units of a resort is modeled by
$$P'(x) = 250e^{1-0.01x^2}$$
dollars per unit rented. At one point, 10 units are rented. What is the net-change in profit when 15 units are rented? (Once you setup this problem you can use technology to perform computations.)

NET-CHANGE $= P(15) - P(10) = \int_{10}^{15} P'(x)dx =$

$= \int_{10}^{15} 250e^{1-0.01x^2}dx \approx 743.21$ DOLLARS.

T.I.

3. Without using technology, compute

$$\frac{d}{dx}\int_{2x^3}^{0}\sqrt{t^4+1}\,dt = -\frac{d}{dx}\left[\int_0^{2x^3}\sqrt{t^4+1}\,dt\right] = -\frac{d}{dx}\left[2x^3\right]\cdot\sqrt{(2x^3)^4+1} =$$

FTC

$$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x) \quad \text{AND CHAIN RULE}$$

$$= -6x^2\sqrt{16x^{12}+1}$$

4. Use the Substitution Method to evaluate the following integral:

$$\int_4^9 x^2\sqrt{x-3}\,dx = \int_1^6 x^2 u^{\frac{1}{2}}\,du = \int_1^6 (u^2+6u+9)\,u^{\frac{1}{2}}\,du =$$

$u = x-3 \Rightarrow du = dx \Rightarrow x = u+3 \Rightarrow x^2 = u^2+6u+9$

$x = 4 \to u = 1$; $x = 9 \to u = 6$

$$= \int_1^6 u^{5/2} + 6u^{3/2} + 9u^{1/2}\,du = \left[\frac{2}{7}u^{7/2} + 6\cdot\frac{2}{5}u^{5/2} + 9\frac{2}{3}u^{3/2}\right]_1^6$$

$$= \left[2u^{3/2}\left(\frac{u^2}{7} + \frac{6}{5}u + 3\right)\right]_1^6 = \frac{6444}{35}\sqrt{6} - \frac{304}{35} = \frac{4}{35}\left(1611\sqrt{6} - 76\right)$$

≈ 442.3003

5.Use Integration by Parts to evaluate the following integral:

$\int x\ln(x)\,dx$ $= \frac{1}{2}x^2\ln x - \int \frac{1}{2}x^2 \cdot \frac{1}{x}dx = \frac{1}{2}\left(x^2\ln x - \int x\,dx\right) =$

$u = \ln x \Rightarrow du = \frac{1}{x}dx$

$dv = x\,dx \Rightarrow v = \int dv = \int x\,dx = \frac{x^2}{2}$

$= \frac{1}{2}\left(x^2\ln x - \frac{1}{2}x^2\right) + C = \frac{1}{4}x^2\left(\ln x^2 - 1\right) + C$

6. Use the Partial Fractions Method to evaluate the following integral:

$\int \frac{4x+3}{(4x-3)(x-2)}\,dx$ $= I$

$\frac{4x+3}{(4x-3)(x-2)} = \frac{A}{4x-3} + \frac{B}{x-2} = \frac{A(x-2)+B(4x-3)}{(4x-3)(x-2)} \Rightarrow A(x-2)+B(4x-3) = 4x+3$

$\Rightarrow \begin{cases} x = 2 \Rightarrow 5B = 11 \Rightarrow B = 11/5 \\ x = \frac{3}{4} \Rightarrow -\frac{5}{4}A = 6 \Rightarrow A = -24/5 \end{cases}$ THEN:

$I = \int \frac{11/5}{x-2} - \frac{24/5}{4x-3}\,dx = \frac{11}{5}\ln|x-2| - \frac{6}{5}\ln|4x-3| + C$

$\hookrightarrow u = 4x-3 \Rightarrow dx = \frac{1}{4}du$

$\overset{OR}{=} \ln\sqrt[5]{\frac{|x-2|^{11}}{(4x-3)^6}} + C.$

7. Determine the mean of the probability density function

$$f(x) = \begin{cases} 0, & x < 0 \\ 5e^{-5x}, & x \geq 0 \end{cases}$$

MEAN

$$\mu = \int_{-\infty}^{\infty} x f(x) = \int_0^{\infty} 5xe^{-5x}\,dx = \lim_{b\to\infty} 5\int_0^b xe^{-5x}\,dx =$$

$$= 5\lim_{b\to\infty}\left[-\frac{1}{5}e^{-5x}\left(\frac{1}{5}+x\right)\right]_0^b =$$

BY PARTS: $\int xe^{-5x}\,dx = -\frac{1}{5}xe^{-5x} + \int \frac{1}{5}e^{-5x}\,dx = -\frac{1}{5}e^{-5x}\left(\frac{1}{5}+x\right)$

$u = x \Rightarrow du = dx$

$dv = e^{-5x}dx \Rightarrow v = \int e^{-5x}dx = -\frac{1}{5}e^{-5x}$

$$= \left(\lim_{b\to\infty} -e^{-5b}\left(b+\frac{1}{5}\right)\right) - \left(-e^{0}\left(0+\frac{1}{5}\right)\right) = 0 + \frac{1}{5} = \frac{1}{5}$$

$$\lim_{b\to\infty} e^{-5b}\left(b+\frac{1}{5}\right) = \lim_{b\to\infty}\frac{b+\frac{1}{5}}{e^{5b}} \overset{\frac{\infty}{\infty}}{\underset{HR}{=}} \lim_{b\to\infty}\frac{1}{5e^{5b}} \overset{\frac{1}{\infty}\to}{=} 0$$

THEN $\mu = \frac{1}{5} = .2$

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8. Without using technology, compute

$$\frac{d}{dz}\int_z^3 \sec(4x+7)\,dx = -\frac{d}{dz}\int_3^z \sec(4x+7)\,dx = \text{FTC}$$

$$= -\sec(4z+7)$$

9. Without using technology to integrate, evaluate the following definite integral:

$$\int_{-1}^{1} x(2x-3)^7\,dx = \int_{-5}^{-1} \frac{1}{2}(u+3)\,u^7\,\frac{du}{2} = \frac{1}{4}\int_{-5}^{-1} u^8 + 3u^7\,du =$$

SUBS: $u = 2x-3 \Rightarrow dx = \frac{1}{2}du$

$x = \frac{1}{2}(u+3)$ AND $\left[\begin{array}{l} x=-1 \Rightarrow u=-5 \\ x=1 \Rightarrow u=-1 \end{array}\right]$

$$= \frac{1}{4}\left[\frac{1}{9}u^9 + \frac{3}{8}u^8\right]_{-5}^{-1} = \frac{1}{4}\left(\frac{5^9}{9} - \frac{3}{8}5^8 - \frac{1}{9} + \frac{3}{8}\right)$$

$$= 17632.\overline{4} = \frac{158692}{9}$$

10. Without using technology to integrate, evaluate the following integral:

$$\int_1^2 \frac{x}{\sqrt{x-1}}dx = \lim_{a \to 1^+} \int_a^2 \frac{x}{\sqrt{x-1}}dx = \lim_{t \to 0^+} \int_t^1 \frac{u+1}{u^{\frac{1}{2}}} du =$$

$u = x - 1 \Rightarrow dx = du$

$x = u + 1$ AND $\left[\begin{array}{l} x \to 1^+ \Rightarrow u \to 0^+ \\ x = 2 \Rightarrow u = 1 \end{array}\right.$

$$= \lim_{t \to 0^+} \int_t^1 u^{\frac{1}{2}} + u^{-\frac{1}{2}} du =$$

$$= \lim_{t \to 0^+} \left[\frac{2}{3}u^{3/2} + 2u^{\frac{1}{2}} \right]_t^1 = \left(\frac{2}{3} + 2\right) - \lim_{t \to 0^+} \underbrace{\left(2t^{\frac{1}{2}}\left(\frac{1}{3}t + 1\right)\right)}_{0 \cdot 1 = 0}$$

$$= \frac{8}{3}$$