MAT 200 - Test 2 - Fall 2015

Instructor: Dr. Francesco Strazzullo **Name** KEY

Instructions. Technology is allowed on this exam, without internet connectivity. Each problem is worth 10 points, for a total of 140 points. You might use the formulas sheet from our book or from our Eagleweb page: **if you do use one cite it. You cannot use cheat-sheets that include solved exercises.** When using technology write down the commands (or keys typed) you used: for instance something like

(*a*) *NORMCDF*(L, R, m, s) (*b*) *NORMCDF*(L, R) (*c*) *NORMCDF*(-1E99, z)

(*d*) *invNORM*(p, m, s) (*e*) *BINOMCDF*(n, p, x) (*f*) *BINOMPDF*(n, p, x)

FOR (a) TO (f) TYPE: [2ND] + [VARS].

SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

1. Consider the following data: DISCRETE RANDOM VARIABLE → GROUPED DATA

NOTE: $\sum P(X=x) = 1$

x	−5	−4	−3	−2	−1
$P(X = x)$	0.2	0.3	0.2	0.1	0.2

→ L_1 → L_2

[STAT] → CALC → 1-VAR. WE GET 1-VAR STATS (L_1, L_2)

a) Find the expected value $E(X)$. Round your answer to one decimal place.

$E(X) = \mu = \sum x \cdot P(X=x) = -3.2$

b) Find the standard deviation. Round your answer to one decimal place.

$\sigma_X = 1.4$

c) Find the variance. Round your answer to one decimal place.

VARS → STAT. → σ_X THEN $V = \sigma_X^2 = 2$ (WAS 1.96)

d) Find the value of $P(X \geq -2)$. Round your answer to one decimal place.

$P(X \geq -2) = P(X=-2) + P(X=-1) = .1 + .2 = .3$

$= 30\%$

e) Find the value of $P(X > -4)$. Round your answer to one decimal place.

$P(X > -4) = 1 - P(X \leq -4) = 1 - (P(X=-4) + P(X=-5))$

↑ ADDITION LAW

$= 1 - (.3 + .2) = 1 - .5 = .5$

$= 50\%$

2. In the long run, which plan has the higher payout? → EXPECTED VALUE

LIKE GROUPED DATA (SEE #1)

NOTE: $\sum P(\text{PAYOUT}) = 1$

Plan A	
Payout	***P*(Payout)**
\$25,000	0.33
\$30,000	0.29
\$70,000	0.38

Plan B	
Payout	***P*(Payout)**
−\$45,000	0.12
−\$30,000	0.44
\$95,000	0.44

L_1 L_2 L_1 L_2

PAYOUT $= X$

$P(\text{PAYOUT}) = P(X = x)$

$E_A(X) = \mu_A = \sum x P_A(X = x) = 25000(.33) + 30000(.29) + 70000(.38)$

$= 43{,}550$ DOLLARS

$E_B(X) = \mu_B = (-45000)(.12) + (-30000)(.44) + 95000(.44)$

$= 23{,}200$ DOLLARS

PLAN A HAS THE HIGHEST PAYOUT.

3. The Magazine Mass Marketing Company has received 15 entries in its latest sweepstakes. They know that the probability of receiving a magazine subscription order with an entry form is 0.55. What is the probability that no less than 6 of the entry forms will include an order? (Round your answer to 3 decimal places)

THIS IS BINOMIAL DISTRIBUTION, WHERE A SUCCESS CORRESPONDS TO RECEIVING A SUBSCRIPTION ORDER. $n = 15$, $p = .55$, $x = 6$

→ $P(X \geq 6) = 1 - P(X < 6) = 1 - P(X \leq 5)$

$P(X \leq 5) = \text{BINOMCDF}(15, .55, 5)$

↓ LEFT-TAIL

→ $P(X \geq 6) = .9231 = 92.31\%$

4. Assume the random variable X has a binomial distribution with the given probability of obtaining a success. Find the following probability, given the sample size. Round your answer to 3 decimal places.

$$P(X = 11), n = 18, p = 0.8$$

$P(X=11) = \text{BINOMPDF}(18, .8, 11) \approx .035 = 3.5\%$

NOT A LEFT-TAIL

5. Assume the random variable X has a binomial distribution with the given probability of obtaining a success. Find the following probability, given the sample size. Round your answer to 3 decimal places.

$$P(X < 5), n = 12, p = 0.4$$

LEFT-TAIL: $P(X<5) = P(X \leq 4) = \text{BINOMCDF}(12, .4, 4)$

$= .438 = 43.8\%$

6. Assume the random variable X has a binomial distribution with the given probability of obtaining a success. Find the following probability, given the sample size. Round your answer to 3 decimal places.

$$P(X \geq 4), n = 10, p = 0.3$$

RIGHT-TAIL: $P(X \geq 4) = 1 - P(X < 4) = 1 - P(X \leq 3)$

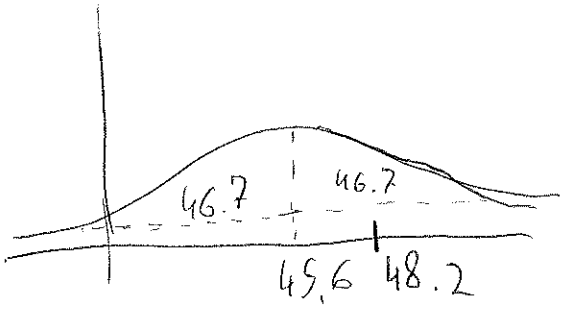
$P(X \leq 3) = \text{BINOMCDF}(10, .3, 3)$

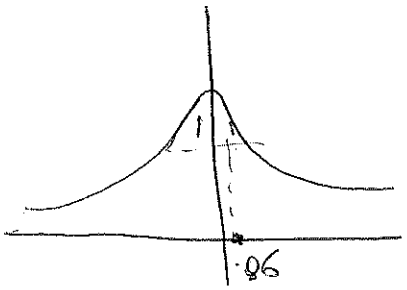
$\rightarrow P(X \geq 4) = .35 = 35\%$

7. Calculate the standard score of the given X value, $X = 48.2$, where $\mu = 45.6$, $\sigma = 46.7$. **Sketch a graph** and round your answer to two decimal places.

$$Z = \frac{X-\mu}{\sigma}$$

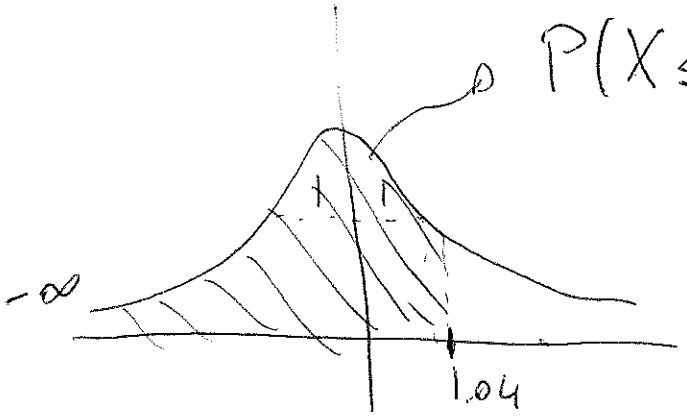
STANDARD





$$Z = \frac{48.2-45.6}{46.7} \approx .06$$

8. Sketch a graph and find the area under the standard normal curve to the left of $z = 1.04$.

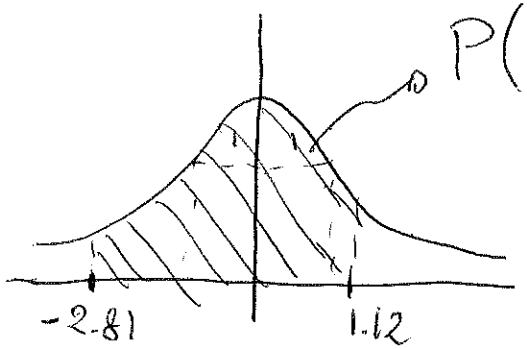


$$P(X \leq z) = P(Z \leq 1.04) = P(-\infty < Z \leq 1.04)$$

$$= \text{NORMALCDF}(-1E99, 1.04)$$

$$= .8508$$

9. Sketch a graph and find the area under the normal curve between $z = -2.81$ and $z = 1.12$.



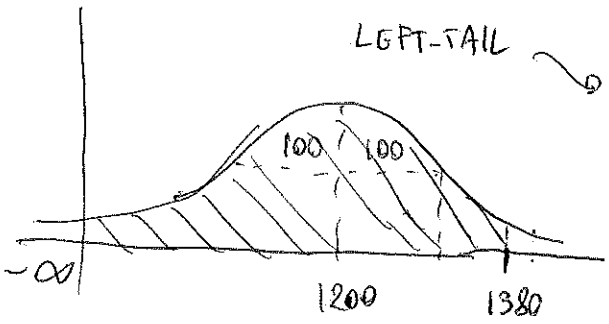
$P(-2.81 \leq Z \leq 1.12) =$

$=$ NORMALCDF $(-2.81, 1.12)$

$= .8662$

10. The weights of steers in a herd are distributed normally. The standard deviation is 100 lbs and the mean steer weight is 1200 lbs. Find the probability that the weight of a randomly selected steer is less than 1380 lbs. (**Sketch a graph** and round your answer to 4 decimal places)

DISTRIBUTED NORMALLY



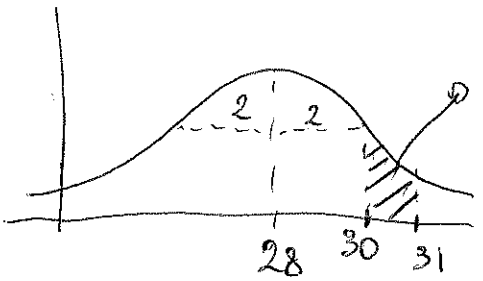
$P(X \leq 1380) = P(X \leq 1380)$

$=$ NORMALCDF $(-1E99, 1380, 1200, 100)$

$= .9641$

$= 96.41\%$

11. A soft drink machine outputs a mean of 28 ounces per cup. The machine's output is normally distributed with a standard deviation of 2 ounces. What is the probability of filling a cup between 30 and 31 ounces? (**Sketch a graph** and round your answer to 4 decimal places)



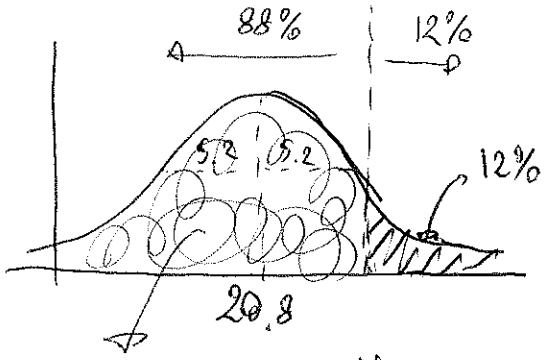
$P(30 \le X \le 31) =$

$= \text{NORMALCDF}(30, 31, 28, 2)$

$= .0918$

$= 9.18\%$

12. Suppose ACT Science scores are normally distributed with a mean of 20.8 and a standard deviation of 5.2. A university plans to send letters of recognition to students whose scores are in the top 12%. What is the minimum score required for a letter of recognition? **Sketch a graph** and round your answer to the nearest whole number, if necessary.



PROBABILITY TO DATA POINT

$.12 = P(X \ge x) = 1 - P(X \le x)$

OR $P(X \le x) = .88$

THEN

FOR LEFT-TAIL USE INVNORM

$x = \text{INVNORM}(.88, 20.8, 5.2)$

$= 26.9 \approx 27$

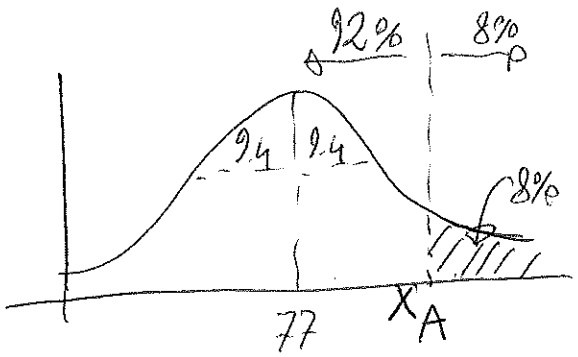
YOU GET A LETTER OF RECOGNITION WITH AN ACT SCIENCE SCORE OF AT LEAST 27.

13. An English professor assigns letter grades on a test according to the following scheme.

A: Top 8% of scores
B: Scores below the top 8% and above the bottom 63%
C: Scores below the top 37% and above the bottom 15%
D: Scores below the top 85% and above the bottom 7%
F: Bottom 7% of scores

Scores on the test are normally distributed with a mean of 77 and a standard deviation of 9.4.

a) (*10 points*) Find the minimum score required for an A grade. **Sketch a graph** and round your answer to the nearest whole number, if necessary.



$8\% = P(X \geq X_A) = 1 - P(X \leq X_A)$

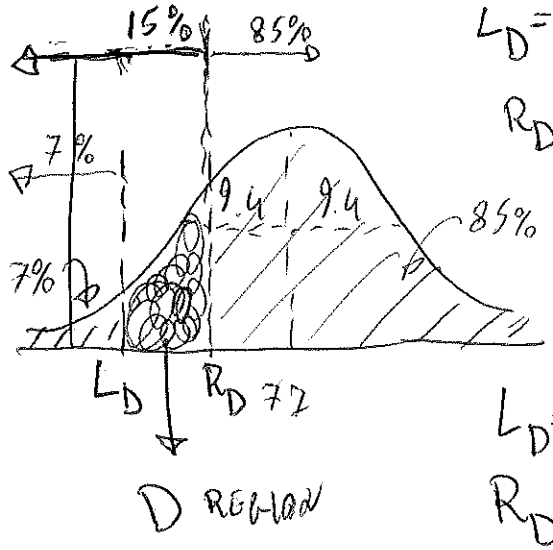
OR $P(X \leq X_A) = 92\% = .92$

THEN

$X_A = \text{INVNORM}(.92, 77, 9.4) = 90.2 \approx 90$

TO GET AN A YOU NEED AT LEAST 90.

b) (*10 points*) Find the numerical limits for a D grade. **Sketch a graph** and round your answers to the nearest whole number, if necessary.



L_D = LOWER VALUE CORRESPONDING TO D = LEFT-POINT

R_D = UPPER VALUE CORRESP. TO D = RIGHT-POINT

$.07 = P(X \leq L_D)$

$.15 = P(X \leq R_D)$

$L_D = \text{INVNORM}(.07, 77, 9.4) = 63.1 \approx 63$

$R_D = \text{INVNORM}(.15, 77, 9.4) = 67.3 \approx 67$

A STUDENT WILL GET A D WITH A SCORE BETWEEN 63 AND 67.