Math 102 - Fall 2009 - Test 3

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Instructions. Only calculators are allowed on this examination. **Each problem is 10 points worth, unless otherwise specified.**

Always use the appropriate wording and units of measure in your answers (when applicable). You might need the following formulas:

$$S = P\left(1+\frac{r}{k}\right)^{kt}, \quad S = P(1+i)^n, \quad S = Pe^{rt}, \quad S = \frac{R}{i}\left[(1+i)^n - 1\right], \quad A = \frac{R}{i}\left[1-(1+i)^{-n}\right].$$

SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

1. Find the future value of \$80,000 invested for 10 years at 5%, compounded annually.

WE MUST USE $S = P\left(1+\frac{r}{k}\right)^{kt}$ FOR $t=10$, $k=1$, $r=.05$

$P = 80,000$ THEN

$S = 80,000\left(1+\frac{.05}{1}\right)^{1\cdot 10} = 130,311.57$

THE FUTURE VALUE OF THIS INVESTMENT IS \$ 130,311.57

2. Without using a calculator, find the value of the following logarithms.

(a) $\log_9 81 = \log_9 9^2 = 2\log_9 9 = 2\cdot 1 = 2$

(b) $\log_3 \frac{1}{27} = \log_3 27^{-1} = -\log_3 27 = -\log_3 3^3 = -3\log_3 3$

$= -3\cdot 1 = -3$

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3. Rewrite the following expression as a single logarithm:

$3\log_2 x + \log_2(x+1)$ $= \log_2 x^3 + \log_2 (x+1)$

$= \log_2 [x^3 (x+1)]$

4. Use a calculator to find the value of the following logarithms.

(a) $\log_2 6 \approx 2.585$

(b) $\log_{1.05} 3.2 \approx 23.84$

5. (15 points) The supply function for a certain size boat is given by $p = 340(2^q)$, where p dollars is the price per boat and q is the quantity of boats supplied at that price. What quantity will be supplied if the price is \$10,880 per boat?

$p = 10,880$, THEN WE MUST SOLVE THE EQUATION.

$340\, 2^q = 10,880 \rightarrow 2^q = \frac{10,880}{340} \rightarrow 2^q = 32$

$\log_2(2^q) = \log_2 32 \quad \rightarrow q = \log_2 2^5 = 5 \rightarrow$

FIVE BOATS ARE SUPPLIED WHEN THEIR PRICE IS \$10,880.

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6. The consumer price index (CPI) is calculated by averaging the prices of various items after assigning a weight to each of them. The following table gives the CPI for selected years from 1940 to 2005, reflecting buying patterns of all urban consumers.

Year	1940	1950	1960	1970	1980
CPI	14	24.1	29.6	38.8	82.4
Year	1990	2000	2002	2004	2005
CPI	130.7	172.2	179.9	188.9	195.3

(a) Using your calculator, find the exponential model which is the best fit for the data. Consider x to be the number of years past 1940 and y to be the CPI. Report your answer to 3 decimal places.

IN 1940 WE HAVE $X=0$, SO THAT IN 2005 $X=65$.

$Y = 14.104(1.042)^X$

(b) Use this model to predict the CPI in 2013.

IN 2013 WE HAVE $X = 2013-1940 = 73$

$Y(73) = 284.24$

(c) According to this model, during what year will the CPI pass 300?

WE COULD USE OUR CALCULATOR (WITH THE $\boxed{\text{TABLE}}$ KEY FOR INSTANCE)

$300 = 14.104(1.042)^X \Rightarrow \frac{300}{14.104} = (1.042)^X \Rightarrow (\log_{1.042})$

$\Rightarrow \log_{1.042}\left(\frac{300}{14.104}\right) = \log_{1.042}(1.042^X) = X \Rightarrow$

$\Rightarrow X = \frac{\log\left(\frac{300}{14.104}\right)}{\log 1.042} = 74.3 \Rightarrow$ DURING $1940+74 = 2014$

BEFORE YEAR 2015

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7. (15 points) Find the future value in 10 years of an investment of \$10,000 at 6% annual interest rate in the following cases.

   (a) Interest compounded monthly.

$$S = P\left(1+\frac{r}{k}\right)^{kt}, \quad r = .06, \quad t = 10, \quad k = 12, \quad P = 10{,}000$$

$$S = 10{,}000\left(1+\frac{.06}{12}\right)^{12\cdot 10} = \$\,18193.98$$

   (b) Interest compounded bimonthly.

AS ABOVE, BUT $k = 2\cdot 12 = 24$

$$S = 10{,}000\left(1+\frac{.06}{24}\right)^{24\cdot 10} = \$\,18{,}207.55$$

   (c) Interest continuously compounded.

WE MUST USE

$$S = P e^{rt}, \text{ THEN:}$$

$$S = 10{,}000\, e^{.06\cdot 10} = \$\,18{,}221.19$$

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8. The winner of a "million dollar" lottery is to receive \$50,000 plus \$50,000 at the end of each year for 19 years, or the present value of this annuity in cash. How much cash would she receive if money is worth 8% compounded annually?

PRESENT VALUE OF ANNUITY. $A = \frac{R}{i}\left[1-(1+i)^{-n}\right]$

HERE: $R = 50,000$, $r = .08$, $t = 19$, $k = 1$. SO THAT $i = .08$, $n = 19$

$$A = \frac{50000}{.08}\left[1-(1+.08)^{-19}\right] = \$480,179.96$$

SINCE 50,000 ARE GIVEN AT THE BEGINNING OF THE FIRST YEAR, THE TOTAL VALUE IS $A + 50,000 = \$530,179.96$

9. To start a new business Beth deposits \$1000 at the end of each six-month period in an account that pays 7% compounded semiannually. How much will she have at the end of 6 years?

IT IS ASKED TO COMPUTE THE FUTURE VALUE OF A STANDARD ANNUITY.

$S = \frac{R}{i}\left[(1+i)^{n}-1\right]$ (TWO PAYMENTS A YEAR)

HERE: $R = 1000$, $r = .07$, $k = 2$, $t = 6$. SO THAT

$i = \frac{r}{k} = \frac{.07}{2} = .035$ AND $n = k \cdot t = 2 \cdot 6 = 12$.

$$S = \frac{1000}{.035}\left((1+.035)^{12}-1\right) = \$14,601.96$$

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