Math 321- Spring 2013 - Exam2 Sample

Instructor: Dr. Francesco Strazzullo Name______________________________

Instructions. Technology and notes (including the formula sheets from our book) are allowed on this exam. Each problem is worth 10 points. **If you use notes or formula sheets, make a reference. When using technology describe which commands (or keys typed) you used or print out your worksheet.**

1. A bacteria culture starts with 350 bacteria and in 75 minutes contains 700 bacteria.
   (a) Use a natural exponential growth ODE to model this culture.
   (b) How many hours does it take to reach 14000 bacteria?

2. Check if the function $y = 3x - 1 + 2e^{-x}$ is a solution of the IVP $\begin{cases} y'' - y' - 2y = 5 - 6x \\ y(0) = -1, \quad y'(0) = 1 \end{cases}$.

3. Find the orthogonal trajectories of the family of curves $y = kx^2$. Then draw several members of each family on the same coordinate plane. (You can attach a printout or upload it in EagleWeb.)

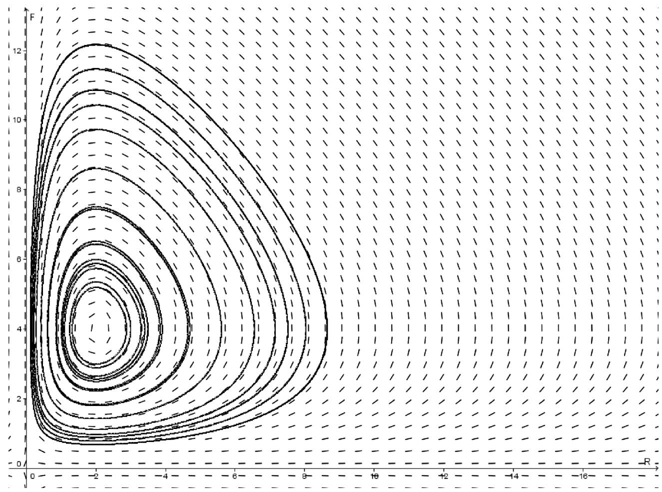
4. Suppose a population growth is modeled by the logistic differential equation with the carrying capacity 2000 and the relative growth rate $k = 0.06$ per year.

   (a) Express the logistic equation.

   (b) Express the general solution.

   (c) Express the particular solution for which $P(0) = 500$.

5. A phase portrait of a predator-prey system is given below in which F represents the population of foxes (in thousands) and R the population of rabbits (in thousands).



(a) Referring to the graph, what is a reasonable non-zero equilibrium solution for the system?

(b) Write down a possible system of differential equations which could have been used to produce the given graph.

6. Consider the following predator-prey system where x and y are in millions of creatures and t represents time in years:

$$\begin{cases} \dfrac{dx}{dt} = 2x - xy \\ \dfrac{dy}{dt} = -4y + xy \end{cases}$$

(a) Show that $(4, 2)$ is the nonzero equilibrium solution.

(b) Find an expression for $\dfrac{dy}{dx}$.

(c) The direction field for the differential equation at point (6b) is given below. Locate $(4, 2)$ on the graph and sketch a rough phase trajectory through P indicated in the graph.