Math 102 - Fall 2012 - Test 4

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Instructions. Only calculators are allowed on this examination. **Each problem is worth 10 points.** *Always use the appropriate wording and units of measure in your answers (when applicable).* You might need the following formulas:

$$A = \frac{R}{i}\left[1-(1+i)^{-n}\right], \quad S = Pe^{rt}, \quad S = \frac{R}{i}\left[(1+i)^n - 1\right], \quad S = P\left(1+\frac{r}{k}\right)^{kt}, \quad S = P(1+i)^n.$$

SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

1. Solve the following logarithmic equation and check any solution.

$\log_6(5+2x) + \log_6(3x-1) = 2$ $\qquad \log_a(MN) = \log_a M + \log_a N$

$\log_6\left((5+2x)(3x-1)\right) = 2$ $\qquad a^{\log_a M} = M$

$(5+2x)(3x-1) = 6^2 \to 15x - 5 + 6x^2 - 2x = 36 \to 6x^2 + 13x - 41 = 0$

SOLVE BY GRAPH  OR QUAD FORMULA $x = \frac{-13 \pm \sqrt{13^2 - 4(6)(-41)}}{2(6)} = \frac{-13 \pm \sqrt{1153}}{12} \approx 1.75,\ -3.91$

CHECK: PLUG IN ORIGINAL EQUATION

I) $x = 1.75 \to \log_6(5+2(1.75)) + \log_6(3(1.75)-1)$ IS DEFINED ✓

II) $x = -3.91 \to \log_6(5+(2)(-3.91))$ IS NOT DEFINED BECAUSE

$5 + 2(-3.91) < 0 \to x = -3.91$ IS AN EXTRANEOUS SOLUTION

2. Use a calculator to find the value of the following expressions.

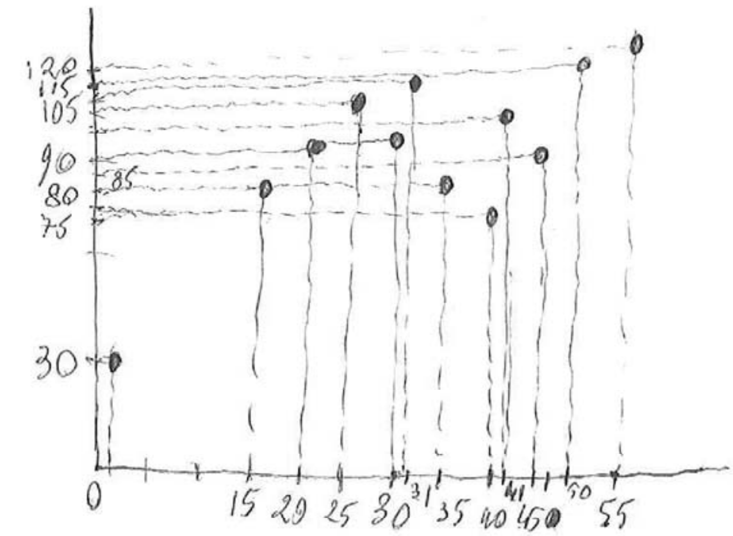
(a) $\log_{1.5}(.75) = \frac{\log(.75)}{\log(1.5)} = -.7095$

(b) $(.6)^{-1.15} = (.6)\wedge(-1.15) = 1.7994$

3. The table below gives the number of new posts on a political blog during the first hour of the last presidential debate, at a selected times (E.T.).

X	Time	Posts	X	Time	Posts	X	Time	Posts
1	9:01 P.M.	30	30	9:30 P.M.	90	41	9:41 P.M.	100
15	9:15 P.M.	80	31	9:31 P.M.	110	45	9:45 P.M.	85
20	9:20 P.M.	90	35	9:35 P.M.	80	50	9:50 P.M.	115
25	9:25 P.M.	105	40	9:40 P.M.	75	55	9:55 P.M.	120

(a) Consider x to be the minutes after the debate started (9:00 P.M.) and y to be the number of new posts, then draw a scatter-plot of the given data.



(b) Using your calculator, find both the quartic and the cubic models which best fit the data. Report your answer to the third decimal place. Report and use the *correlation coefficients* to say which of these models is the best fit for the given data.

CUBIC: $Y = .003X^3 - .274X^2 + 7.961X + 20.398$, $R^2 = .816$

QUARTIC: $Y = .00006X^4 - .003X^3 - .038X^2 + 5.127X + 24.249$, $R^2 = .828$.

THE QUARTIC MODEL IS THE BEST FIT. NOTE THAT THE LEADING COEFFICIENT IS 5.626 E-5 $= 5.626(10^{-5})$.

(c) Use the unrounded best model to compute examples of interpolation and extrapolation from the given data.

INTERPOLATION (INSIDE POINTS): AT 9:10 PM $\rightarrow X = 10$, $Y = 68.8 \approx 69$ NEW POSTS.

EXTRAPOLATION (OUTSIDE POINTS): AT 10:00 PM $\rightarrow X = 60$, $Y = 170.5 \approx 171$ NEW POSTS.

4. Your financial planner is suggesting you two short term investments, which require periodic payments for 10 years as follows:

   (a) \$100 a month at 3.5%, or

   (b) \$25 a week at 3%.

   Compute which option is the best deal for you.

"PERIODIC PAYMENTS" = "CASH FLOW" = ANNUITY.

WE MUST COMPARE THE FUTURE VALUES OF THESE OPTIONS: $S = \frac{R}{i}\left((1+i)^n - 1\right)$

(a) $R = 100$; $r = 3.5\% = 0.035$; $k = 12$; $t = 10 \rightarrow$

$\rightarrow i = \frac{r}{k} = \frac{.035}{12} \approx .0029167$; $n = kt = 12(10) = 120$

$$S = \frac{100}{.035/12}\left(\left(1 + \frac{.035}{12}\right)^{120} - 1\right) = \$\,14{,}343.25$$

"RETURN OF INVESTMENT" $= S - 100(120) = \$2343.25$

(b) $R = 25$; $r = 3\% = .03$; $k = 52$; $t = 10 \rightarrow$

$\rightarrow i = \frac{.03}{52} \approx .0005762$; $n = 520 \rightarrow$

$$\rightarrow S = \frac{25}{.03/52}\left(\left(1 + \frac{.03}{52}\right)^{520} - 1\right) = \$\,15{,}155.49$$

"RETURN OF INVESTMENT" $= S - 25(520) = \$2155.49$

OPTION (a) IS THE BEST DEAL.

5. You can invest \$4500 for the next five years. They offer you two options:

   (a) a continuously compounded account at 1.25%, or

   (b) a daily compounded account at 1.5%.

   Compute which option is the best deal for you.

THESE ARE SIMPLE INVESTMENT OF A LUMP SUM $P = 4500$ OVER $t = 5$ YEARS.

(a) $S = Pe^{rt}$

$r = 1.25\% = 0.0125$ $\rightarrow S = 4500\, e^{.0125(5)} = 4790.22$

(b) $S = P\left(1 + \frac{r}{k}\right)^{kt}$

$r = 1.5\% = .015$; $k = 365$ (EVEN IF THERE IS A LEAP YEAR!) $\rightarrow$

$\rightarrow S = 4500\left(1 + \frac{.015}{365}\right)^{365(5)} = 4850.47$

OPTION (b) IS THE BEST DEAL.