MAT 121 – Exam2 – Spring 2014

Instructor: Dr. Francesco Strazzullo **Name** KEY

70 PTS + 10 EXTRA

Instructions. Complete 7 out of the following 10 exercises, as indicated. Exercise 11 is for extra points. Each exercise is worth 10 points. If you need to approximate then **round to 3 decimal places**. You can use a graphing tool and/or a computer algebra system like GeoGebra. When solving a problem graphically sketch the graph you used. **SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).**

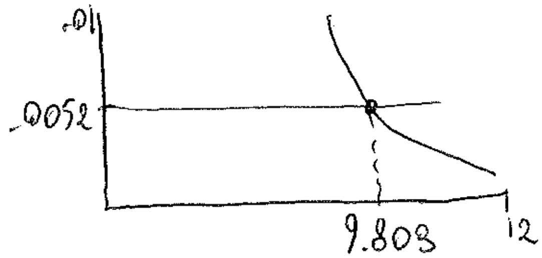
Complete 1 of the exercises 1-2

1. Solve for x: $5^{-x/3} = 0.0052$.

ALGEBRA: $\log_5\left(5^{-x/3}\right) = \log_5\left(\frac{52}{10000}\right) \Rightarrow -\frac{x}{3} = \log_5\left(\frac{13}{2500}\right) \Rightarrow$

$\Rightarrow x = -3\log_5\left(\frac{13}{2500}\right) = \log_5\left(\frac{2500^3}{13^3}\right) \approx \boxed{9.803}$

GRAPH:



A CAS COULD BE USED.

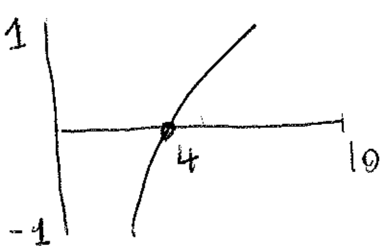
2. Identify the x-intercept of the function $f(x) = 2\ln(x-3)$.

X-INTERCEPT IS A ZERO of $f(x)$: $x = c$ FOR WHICH $f(c) = 0$.

SOLVE: $2\ln(x-3) = 0$.

ALGEBRA: $\ln(x-3) = 0 \Rightarrow e^{\ln(x-3)} = e^0 \Rightarrow x-3 = 1 \Rightarrow \boxed{x = 4}$

GRAPH



OR CAS

$\frac{10}{20}$

Complete 2 of the exercises 3-5

3. Evaluate the function $f(x) = \frac{1}{4}\log_{3.5}(x-1)$ at $x = 2.098$

$f(2.098) = \frac{1}{4}\log_{3.5}(2.098-1) = \frac{1}{4}\log_{3.5}(1.098) \approx .01865683$

$\log_{3.5}(1.098) \underset{\text{GGB}}{=} \log(3.5, 1.098) \underset{\text{TI-84 CHANGE OF BASE}}{=} \log(1.098)/\log(3.5)$

TO 3RD DECIMAL PLACE: $\boxed{f(2.098) = .019}$

4. What is the value of the function $f(x) = 4.8(3^{-1.8x})$ at $x = 2.5$?

$f(2.5) = 4.8\,(3^{-1.8(2.5)}) = 4.8(3^{-4.5}) \approx .034$

5. Identify the value of the function $f(x) = \log(2x-10)$ at $x = 715$.

$f(715) = \log(2(715)-10) = \underset{\text{STANDARD } \log_{10}}{\log}(1420) \approx 3.152$

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Complete both exercises 6-7

6. An initial investment of \$2000 doubles in value in 6.6 years. Assuming continuous compounding, what was the interest rate? Round to the nearest tenth of a percent.

n-TIMES COMPOUNDED SIMPLE INVESTMENT: $A = P\left(1+\frac{r}{n}\right)^{nt}$

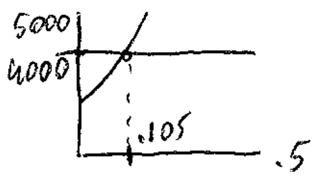
CONTINUOUSLY COMPOUNDED: $A = Pe^{rt}$

P = PRINCIPAL = 2000; FOR $t = 6.6$ IT IS $A = 2(2000) = 4000$.

SOLVE: $4000 = 2000\, e^{r(6.6)}$

ALGEBRA: $2 = e^{6.6r} \Rightarrow \ln(e^{6.6r}) = \ln 2 \Rightarrow 6.6r = \ln 2 \Rightarrow$

$\Rightarrow r = \frac{\ln 2}{6.6} \approx .10502$ AS PERCENT $r \approx 10.5\%$

GRAPHICALLY 

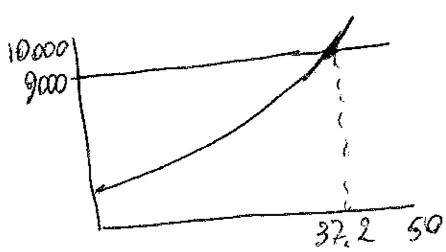
7. The population P of a culture of bacteria is described by the equation $P = 1300e^{0.052t}$, where t is the time, in hours, relative to the time at which the population was 1300.
   (a) What was the population at $t = 4$ hours?
   (b) After how many hours will the population reach 9000? Round to the nearest tenth of an hour.

(a) $P(4) = 1300\, e^{.052(4)} \approx 1600.577 \Rightarrow P(4) = 1600$

(b) SOLVE: $1300\, e^{.052t} = 9000$

ALGEBRA: $e^{.052t} = \frac{9000}{1300} \Rightarrow \ln(e^{.052t}) = \ln\left(\frac{90}{13}\right) \Rightarrow$

$\Rightarrow .052t = \ln\left(\frac{90}{13}\right) \Rightarrow t = \frac{1}{.052}\ln\left(\frac{90}{13}\right) \approx 37.208 \Rightarrow \boxed{t = 37.2}$ HOURS

GRAPH (OR CAS) 

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Complete 1 of the exercises 8-9

8. The pH of an acidic solution is a measure of the concentration of the acid particles in the solution, with smaller values of the pH indicating higher acid concentration. In a laboratory experiment, the pH of a certain acid solution is changed by dissolving over-the-counter antacid tablets into the solution. In this experiment, the pH changes according to the equation

$$\mathrm{pH} = 4.67 + \log\left(\frac{x}{0.5 - x}\right),$$

where x is the number of grams of antacid added to the solution. What is the pH of the solution after the addition of 0.35 grams of antacid tablet?

$$\mathrm{pH}(0.35) = 4.67 + \log\left(\frac{.35}{0.5 - .35}\right) \approx 5.03797$$

WITH .35 GRAMS OF ANTACID THE pH BECOMES 5.038

9. The chemical acidity of a solution is measured in units of pH: $\mathrm{pH} = -\log\left[\mathrm{H}^{+}\right]$, where $\left[\mathrm{H}^{+}\right]$ is the hydrogen ion concentration in the solution. What is $\left[\mathrm{H}^{+}\right]$ if $\mathrm{pH} = 3.8$?

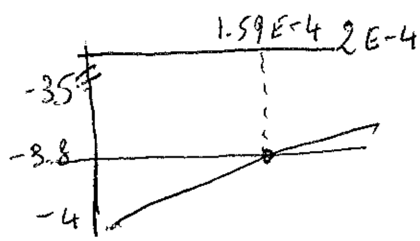
USE $Y = \mathrm{pH}$ AND $X = [H^{+}]$.

SOLVE: $-\log X = 3.8$

ALGEBRA: $\log X = -3.8 \Rightarrow 10^{\log X} = 10^{-3.8} \Rightarrow X = 10^{-3.8}$

TO 3 FIGURES: $X \approx 1.585\ E-4$

GRAPH



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Complete this exercise: you can download the data-sheet from the coursework section in EagleWeb.

10. The table below gives the number of births, in thousands, to females over the age of 35 for a particular state every two years from 1980 to 1996.

Year	1980	1982	1984	1986	1988	1990	1992	1994	1996
Births (thousands)	44.5	36.0	32.1	40.3	46.7	50.8	56.9	51.4	47.2

Consider x to be the number of years after 1980, and y to be the birth. Use technology to answer to the following questions.

(a) Find the cubic and the quartic models that are the best fit for these data. (Round your answer to five decimal places).
(b) Use the correlation coefficients from part (a) to decide which model is better.
(c) Use the unrounded best model from part (b) to estimate how many births to females over the age of 35 there were in this state in 1995. Round to the nearest newborn using the greatest integer function.

(a) CUBIC: $Y = -.05123X^3 + 1.22541X^2 - 6.37619X + 44.02424$

$R^2 = .94832$

QUARTIC: $Y = .00237X^4 - .12693X^3 + 1.97834X^2 - 8.73368X + 44.93263$

$R^2 = .9654$

(b) QUARTIC IS BETTER BECAUSE R^2 IS LARGER.

(c) IN 1995 IT IS $X = 1995 - 1980 = 15 \Rightarrow Y = 50.42338$

$\Rightarrow$ 50,423 BIRTHS.

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Extra points

11. Carbon dating presumes that, as long as a plant or animal is alive, the proportion of its carbon that is ^{14}C is constant. The amount of ^{14}C in an object made from harvested plants, like paper, will decline exponentially according to the equation $A = A_0 e^{-0.0001213t}$, where A represents the amount of ^{14}C in the object, A_o represents the amount of ^{14}C in living organisms, and t is the time in years since the plant was harvested. If an archeological artifact has 25% as much ^{14}C as a living organism, how old would you predict it to be? Round to the nearest year.

WE DON'T KNOW A_0, BUT WE ARE GIVEN $A = 25\% (A_0)$

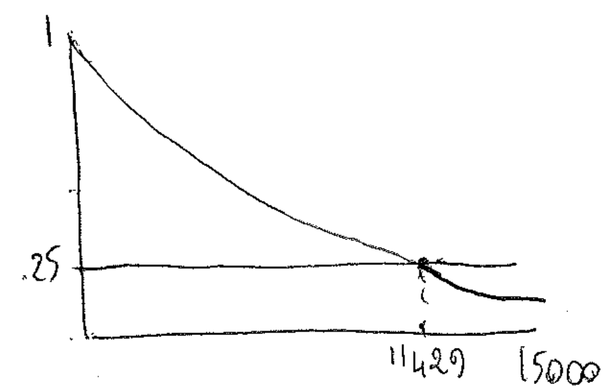
THAT IS $A = .25 A_0$. PLUG IN THE MODEL:

$.25 \cancel{A_0} = \cancel{A_0} e^{-.0001213t}$ AND SOLVE.

ALGEBRA: $\ln(.25) = \ln(e^{-.0001213t}) \Rightarrow -\frac{1213}{10^7} t = \ln(.25)$

$\Rightarrow t = -\frac{10^7}{1213} \ln(.25) \approx 11{,}428.6 \approx 11{,}429.$

GRAPH



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