MAT 121 – Exam3 – Spring 2015

Instructor: Dr. Francesco Strazzullo **Name** KEY

I certify that I did not receive third party help in *completing* this test (sign) ______________

Instructions. Complete 10 out of the following 13 exercises, as indicated. Each exercise is worth 10 points. If you need to approximate then **round to 3 decimal places**, unless otherwise specified. You can use a graphing tool and/or a computer algebra system like GeoGebra. When solving a problem graphically sketch the graph you used.
SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

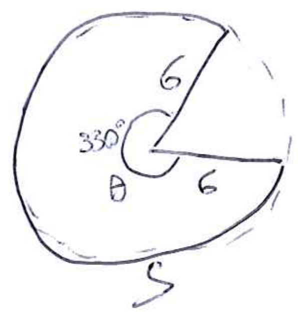
Complete 2 of the exercises 1-3

1. Rewrite 220° in radian measure as a multiple of π.

$$220^\circ = 220 \cdot \frac{\pi}{180} \text{ rad} = \frac{11}{9}\pi \text{ rad}$$

$$\approx 3.84$$

2. Find the length of the arc, S, on a circle of radius 6 meters intercepted by a central angle of 330°. Round to two decimal places.



$$\theta \text{ rad} = \frac{S}{r} \Rightarrow S = r\,\theta$$

$$\theta = 330^\circ = 330 \frac{\pi}{180} \text{ rad} = \frac{11}{6}\pi \text{ rad}$$

$$\Rightarrow S = 6\left(\frac{11}{6}\pi\right) = 11\pi \text{ meters}$$

$$\approx 34.558$$

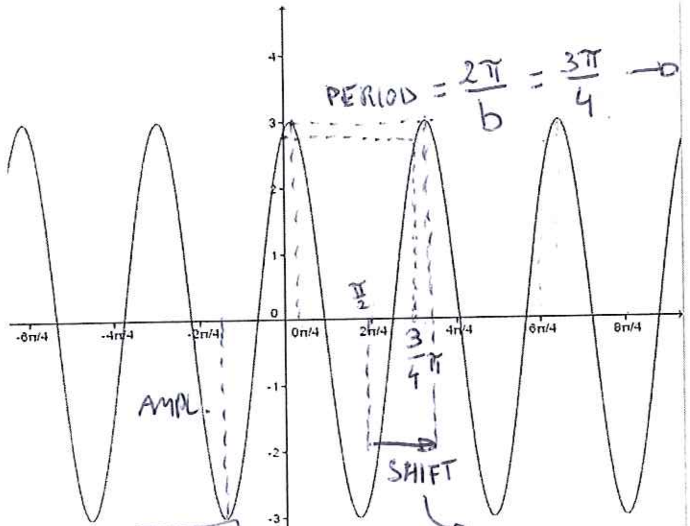
3. Find the area of the sector of the circle with radius 2 centimeters and central angle $\frac{5\pi}{3}$.

$$A = \frac{\theta r^2}{2} = \frac{1}{2}\,\frac{5\pi}{3}(2)^2 = \frac{10}{3}\pi \text{ SQUARE CENTIMETERS.}$$

$$\approx 10.472$$

Complete 2 of the exercises 4-6

4. Find a, b, and c for the function $f(x) = a\ \sin(bx - c)$ such that the graph of $f(x)$ matches the graph below.



$\text{PERIOD} = \frac{2\pi}{b} = \frac{3\pi}{4} \rightarrow \boxed{b = \frac{8}{3} \approx 3}$

$f(x) = 3\sin(3x+1)$

OR

$f(x) = 3\sin\left(\frac{8}{3}x+1\right)$

$\boxed{a = 3}$

$\rightarrow c \approx \frac{\pi}{4} + \text{SOMETHING}$

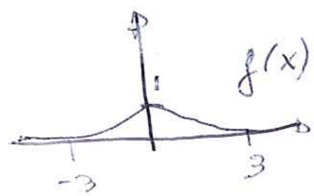
$\boxed{c \approx 1}$

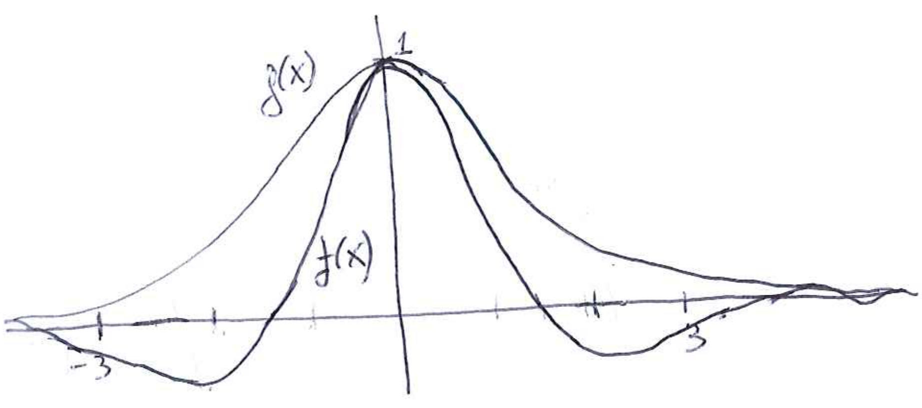
5. Use a graphing utility to **graph in the same viewing window the damping factor and the function**

$$f(x) = e^{\frac{-x^2}{3}} \cos x\ .$$

Sketch the graphs and describe the behavior of the function as x increases without bound.

DAMPING FACTOR: $g(x) = e^{-\frac{x^2}{3}}$





$f(x) \rightarrow 0$ AS $x \rightarrow \infty$

6. Determine at least two solutions of the equation $\sec x = -3$ such that $0 \le x < 2\pi$.

ALGEBRA: $\sec x = -3 \Rightarrow \frac{1}{\cos x} = -3 \Rightarrow \cos x = -\frac{1}{3} \Rightarrow$

$\Rightarrow x = \arccos\left(-\frac{1}{3}\right) <$ $x \approx 1.91 + 2k\pi \xrightarrow{0 \le x < 2\pi} x \approx 1.91$

$x \approx -1.91 + 2k\pi \longrightarrow x \approx -1.91 + 2\pi = 4.32$

GRAPH:

Complete all the exercises 7, 8, and 9

7. Ignoring the effects of air resistance, the range of a projectile fired at an angle θ with the horizontal and with an initial velocity of v_0 feet per second is $r = \frac{1}{32} v_0^2 \sin 2\theta$ where r is measured in feet. A golfer strikes a golf ball at 95 feet per second. At what angle must the golfer hit the ball so that it travels (a) 140 feet? (b) it has the maximum range?

(a) $r = \frac{1}{32}(95)^2 \sin(2\theta) \rightarrow r = \frac{9025}{32} \sin(2\theta)$

$140 = \frac{9025}{32} \sin(2\theta) \Rightarrow \sin(2\theta) = \frac{896}{1805} \Rightarrow 2\theta = \arcsin\left(\frac{896}{1805}\right)$

$\Rightarrow \theta = \frac{1}{2} \arcsin\left(\frac{896}{1805}\right) \approx .26 \text{ rad} = 14.88^\circ$

(b) ONE COULD GRAPH IN THE θ, r - PLANE, ANYWAY

THE MAXIMUM RANGE IS AT MAXIMUM FOR $\sin(2\theta)$

THAT IS $\sin(2\theta) = 1 \Rightarrow 2\theta = \frac{\pi}{2} \Rightarrow \boxed{\theta = \frac{\pi}{4} = 45^\circ}$

8. A Ferris wheel is built such that the height h (in feet) above the ground of a seat on the wheel at time t (in seconds) can be modeled by $h(t) = 70 + 53\sin\left(\frac{\pi}{24}t - \frac{\pi}{2}\right)$. The wheel makes one revolution every 48 seconds and the ride begins when $t = 0$. During the first 48 seconds of the ride, when will a person, who starts at the bottom of the Ferris wheel, be (a) 70 feet above the ground? (b) at the maximum height?

BOTTOM OF FERRIS WHEEL $\to h(0) = 0$.

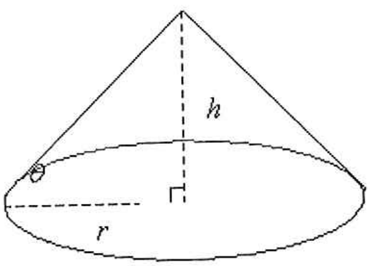
(a) $h(t) = 70 \Rightarrow 53\sin\left(\frac{\pi}{24}t - \frac{\pi}{2}\right) = 0 \Rightarrow \frac{\pi}{24}t - \frac{\pi}{2} = 0$ OR

$\frac{\pi}{24}t - \frac{\pi}{2} = \pi \Rightarrow t = 12$ OR $t = \frac{3}{2} \cdot 24 = 36$ SECONDS

(b) MAXIMUM (GRAPH OR) WHEN $\sin\left(\frac{\pi}{24}t - \frac{\pi}{2}\right) = 1 \Rightarrow$

$\Rightarrow \frac{\pi}{24}t - \frac{\pi}{2} = \frac{\pi}{2} \Rightarrow t = 24$ SECONDS

9. A granular substance such as sand naturally settles into a cone-shaped pile when poured from a small aperture. Its height depends on the humidity and adhesion between granules. The angle of elevation of a pile, θ, is called the angle of repose. If the height of a pile of sand is 14 feet and its diameter is approximately 47 feet, determine the angle of repose. Round your answer to the nearest degree.



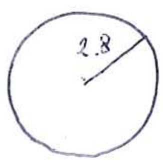
$\frac{h}{r} = \tan(\theta) \Rightarrow \theta = \arctan\left(\frac{h}{r}\right) \Rightarrow$

$\Rightarrow \theta = \arctan\left(\frac{14}{47/2}\right) \approx .54 \text{ rad} = 31^\circ$

NOTE: $2r = 47$

Complete 1 of the exercises 10-11

10. A car is traveling along Route 66 at a rate of 65 miles per hour, and the diameter of its wheels are 2.8 feet. Find the number of revolutions per minute the wheels are turning. Round your answer to one decimal place.



RATE $= 65 \frac{\text{mile}}{\text{hour}} = 65 \frac{5280 \text{ FT}}{60 \text{ min}} = 5720 \text{ FT/min}$

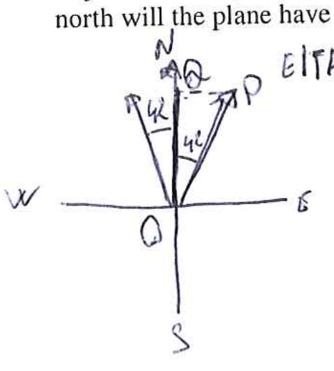
1 REVOLUTION = CIRCUMFERENCE $= \pi(2.8)$

REVOLUTIONS $= \frac{\text{RATE}}{\text{CIRC.}} = \frac{5720}{2.8\pi} \approx$ 650.3 PER MINUTE

NOTE: TIME $= \frac{\text{DISTANCE}}{\text{RATE}}$

FREQUENCY $= \frac{1}{\text{TIME}} = \frac{\text{RATE}}{\text{DISTANCE}}$

11. A jet is traveling at 610 miles per hour at a bearing of 42°. After flying for 1.6 hours in the same direction, how far north will the plane have traveled? Round your answer to the nearest mile.



EITHER

$\overline{OQ} = \overline{OP} \cos 42$

$\overline{OP} =$ SPEED · TIME $= 610 \cdot 1.6 = 976$ miles

$\overline{OQ} = 976 \cdot \cos 42 \approx 725$ MILES

Complete both exercises 12 and 13: you can download the data-sheet for exercise 12 from the coursework section in EagleWeb.

12. The table shows the mean monthly temperature T (in degrees Fahrenheit) and the mean monthly precipitation P (in inches) for Honolulu (Hawaii) where t is the month, with $t = 1$ corresponding to January. (Data Source: National Climatic Data Center)

Month, t	1	2	3	4	5	6	7	8	9	10	11	12
T	71	71	73	74	78	81	82	84	84	81	77	73
P	2.5	2.3	1.7	0.9	0.7	0.3	0.6	0.6	0.8	2.4	2.5	3.1

(a) Use the sine regression feature of a graphing utility to find sine models to fit each set of data. Report these models and the corresponding correlation coefficients. $Y = K + a \sin(bx - c)$
(b) Which data is best fit by its sine regression?
(c) What is the period of each model?
(d) What is the amplitude of each model? Interpret the meaning of the amplitude for each model in the context of the problem.
(e) At what values of t does each sine model reach its maximum and minimum? What do these values represent in the context of the problem?

(a) $T(t) = 77.338 + 6.608 \sin(.515t - 2.508)$, $R^2 = .9294$

$P(t) = 1.763 + 1.435 \sin(.449t + 1.910)$, $R^2 = .93721$

(b) T IS BEST FIT.

(c) PERIOD $= \frac{2\pi}{b}$ <
T: $\frac{2\pi}{.515} \approx 12.2$ MONTHS ⇒ 1 YEAR
P: $\frac{2\pi}{.449} \approx 13.99 \approx 14$ MONTHS

(d) NOTE: K IS THE MEDIAN VALUE, $|a|$ IS THE AMPLITUTE, $K-a$ MIN. VALUE, AND $K+a$ MAX VALUE.

T: $|a| = 6.608$ ⇒ THE TEMPERATURE HAS A $2|a| \approx 13\,°F$ SWING FROM MAX TO MIN

P: $|a| = 1.435$ → THE PRECIPITATION HAS A $2|a| \approx 2.9$ INCHES SWING FROM LOW TO HIGH.

(e) MAX AT $bx - c = \frac{\pi}{2}$ AND MIN AT $bx - c = \frac{3}{2}\pi$ OR $bx - c = -\frac{\pi}{2}$

T: $.515t - 2.508 = \frac{\pi}{2} \to t \approx 7.92$ → END OF JULY (HOTTEST MONTH)

$.515 - 2.508 = -\frac{\pi}{2} \to t \approx 1.82$ → END OF JANUARY (COOLEST MONTH.)

P: $.449t + 1.910 = \frac{\pi}{2} \to t \approx -.755$ → BEFORE JAN., AT THE BEGINNING OF DEC. (WETTEST)

$.449t + 1.910 = \frac{3}{2}\pi \to t \approx 6.24$ → BEGINNING OF JUNE IS THE DRIEST.

13. Use a graphing utility to approximate (to three decimal places) all the solutions of the given equation in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

$$3\tan^4 x - 19\tan^2 x = -2$$

$Y_1 = 3(\tan x)^4 - 19(\tan x)$

$Y_2 = -2$

ALGEBRA:

$Z = (\tan x)^2 \Rightarrow 3Z^2 - 19Z + 2 = 0 \Rightarrow Z = \dfrac{19 \pm \sqrt{19^2 - 24}}{6} = \dfrac{19 \pm \sqrt{337}}{6}$

THEN $\tan x = \pm\sqrt{\dfrac{19 \pm \sqrt{337}}{6}} \Rightarrow x = \arctan\left(\pm\sqrt{\dfrac{19 \pm \sqrt{337}}{6}}\right)$

$\approx \pm 1.19, \pm .32$