Math 102 - Fall 2009 - Test 3

Instructor: Dr. Francesco Strazzullo Name: KEY

Instructions. Only calculators are allowed on this examination. **Each problem is 10 points worth, unless otherwise specified.**
Always use the appropriate wording and units of measure in your answers (when applicable). You might need the following formulas:

$$S = P\left(1+\frac{r}{k}\right)^{kt}, \quad S = P(1+i)^n, \quad S = Pe^{rt}, \quad S = \frac{R}{i}\left[(1+i)^n - 1\right], \quad A = \frac{R}{i}\left[1-(1+i)^{-n}\right].$$

SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

1. Find the future value of \$80,000 invested for 10 years at 5%, compounded annually.

WE MUST USE $S = P\left(1+\frac{r}{k}\right)^{kt}$ FOR $t=10$, $k=1$, $r=.05$
$P=80,000$ THEN

$$S = 80,000\left(1+\frac{.05}{1}\right)^{1\cdot 10} = 130,311.57$$

THE FUTURE VALUE OF THIS INVESTMENT IS \$ 130,311.57

2. Without using a calculator, find the value of the following logarithms.

   (a) $\log_9 81 = \log_9 9^2 = 2\log_9 9 = 2\cdot 1 = 2$

   (b) $\log_3 \frac{1}{27} = \log_3 27^{-1} = -\log_3 27 = -\log_3 3^3 = -3\log_3 3$
   $= -3\cdot 1 = -3$

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3. Rewrite the following expression as a single logarithm:

$3\log_2 x + \log_2(x+1)$ $= \log_2 x^3 + \log_2 (x+1)$

$= \log_2 [x^3 (x+1)]$

4. Use a calculator to find the value of the following logarithms.

(a) $\log_2 6 \approx 2.585$

(b) $\log_{1.05} 3.2 \approx 23.84$

5. (15 points) The supply function for a certain size boat is given by $p = 340(2^q)$, where p dollars is the price per boat and q is the quantity of boats supplied at that price. What quantity will be supplied if the price is \$10,880 per boat?

$p = 10,880$, THEN WE MUST SOLVE THE EQUATION.

$340\, 2^q = 10,880 \rightarrow 2^q = \frac{10,880}{340} \rightarrow 2^q = 32$

$\log_2(2^q) = \log_2 32 \rightarrow q = \log_2 2^5 = 5 \rightarrow$

FIVE BOATS ARE SUPPLIED WHEN THEIR PRICE IS \$10,880.

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6. The consumer price index (CPI) is calculated by averaging the prices of various items after assigning a weight to each of them. The following table gives the CPI for selected years from 1940 to 2005, reflecting buying patterns of all urban consumers.

Year	1940	1950	1960	1970	1980
CPI	14	24.1	29.6	38.8	82.4
Year	1990	2000	2002	2004	2005
CPI	130.7	172.2	179.9	188.9	195.3

(a) Using your calculator, find the exponential model which is the best fit for the data. Consider x to be the number of years past 1940 and y to be the CPI. Report your answer to 3 decimal places.

IN 1940 WE HAVE $X=0$, SO THAT IN 2005 $X=65$.

$Y = 14.104(1.042)^X$

(b) Use this model to predict the CPI in 2013.

IN 2013 WE HAVE $X = 2013-1940 = 73$

$Y(73) = 284.24$

(c) According to this model, during what year will the CPI pass 300?

WE COULD USE OUR CALCULATOR (WITH THE $\boxed{\text{TABLE}}$ KEY FOR INSTANCE)

$300 = 14.104(1.042)^X \Rightarrow \frac{300}{14.104} = (1.042)^X \Rightarrow (\log_{1.042})$

$\Rightarrow \log_{1.042}\left(\frac{300}{14.104}\right) = \log_{1.042}(1.042^X) = X \Rightarrow$

$\Rightarrow X = \frac{\log\left(\frac{300}{14.104}\right)}{\log 1.042} = 74.3 \Rightarrow$ DURING $1940+74 = 2014$

BEFORE YEAR 2015

10

7. (15 points) Find the future value in 10 years of an investment of $10,000 at 6% annual interest rate in the following cases.

(a) Interest compounded monthly.

$S = P\left(1+\frac{r}{k}\right)^{kt}$, $r = .06$, $t = 10$, $k = 12$, $P = 10{,}000$

$S = 10{,}000\left(1+\frac{.06}{12}\right)^{12 \cdot 10} = \$\,18193.98$

(b) Interest compounded bimonthly.

AS ABOVE, BUT $k = 2 \cdot 12 = 24$

$S = 10{,}000\left(1+\frac{.06}{24}\right)^{24 \cdot 10} = \$\,18{,}207.55$

(c) Interest continuously compounded.

WE MUST USE

$S = Pe^{rt}$, THEN:

$S = 10{,}000\, e^{.06 \cdot 10} = \$18{,}221.19$

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8. The winner of a "million dollar" lottery is to receive $50,000 plus $50,000 at the end of each year for 19 years, or the present value of this annuity in cash. How much cash would she receive if money is worth 8% compounded annually?

PRESENT VALUE OF ANNUITY. $A = \frac{R}{i}\left[1-(1+i)^{-n}\right]$

HERE: $R = 50{,}000$, $r = .08$, $t = 19$, $k = 1$. SO THAT $i = .08$, $n = 19$

$$A = \frac{50000}{.08}\left[1-(1+.08)^{-19}\right] = \$\,480{,}179.76$$

SINCE 50,000 ARE GIVEN AT THE BEGINNING OF THE FIRST YEAR, THE TOTAL VALUE IS $A + 50{,}000 = \$530{,}179.76$

9. To start a new business Beth deposits $1000 at the end of each six-month period in an account that pays 7% compounded semiannually. How much will she have at the end of 6 years?

IT IS ASKED TO COMPUTE THE FUTURE VALUE OF A STANDARD ANNUITY.

$S = \frac{R}{i}\left[(1+i)^{n}-1\right]$ (TWO PAYMENTS A YEAR)

HERE: $R = 1000$, $r = .07$, $k = 2$, $t = 6$. SO THAT

$i = \frac{r}{k} = \frac{.07}{2} = .035$ AND $n = k \cdot t = 2 \cdot 6 = 12$.

$$S = \frac{1000}{.035}\left((1+.035)^{12}-1\right) = \$\,14{,}601.96$$

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Math 102 - Spring 2010 - Test 3

Instructor: Dr. Francesco Strazzullo Name KEY

Instructions. Only calculators are allowed on this examination. **Each problem is 10 points worth, unless otherwise specified.**

Always use the appropriate wording and units of measure in your answers (when applicable). You might need the following formulas:

$$S = P\left(1 + \frac{r}{k}\right)^{kt}, \quad S = P(1+i)^n, \quad S = Pe^{rt}, \quad S = \frac{R}{i}\left[(1+i)^n - 1\right], \quad A = \frac{R}{i}\left[1 - (1+i)^{-n}\right].$$

SHOW YOUR WORK NEATLY, PLEASE (no work, no credit).

1. Use a calculator to find the following numbers.

   (a) $\log_5 72 = \dfrac{\log 72}{\log 5} \approx 2.657$

   (b) $\log_{0.1} 2.4 = \dfrac{\log 2.4}{\log 0.1} \approx -.38$

   (c) $5^{6/7} \approx 3.973$

2. Rewrite the following expression as a single logarithm:

   $\log_3(1-x) + 4\log_3 x$

   $= \log_3(1-x) + \log_3 x^4$

   $= \log_3\left((1-x)\cdot x^4\right)$

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3. (15 points) Suppose the weekly cost for the production of x units of a product is given by

$$C(x) = 3452 + 50\ln(x+1) \text{ dollars.}$$

Estimate the number of units produced if the weekly cost is \$ 3556.

$C = 3556$ (USE GRAPHIC METHODS $\begin{cases} Y_1 = C(x) \\ Y_2 = 3556 \end{cases}$ OR ALGEBRA)

$$\begin{array}{l} 3556 = 3452 + 50\ln(x+1) \\ -3452 \quad -3452 \end{array} \longrightarrow \frac{104}{50} = \frac{50\ln(x+1)}{50} \longrightarrow$$

$\longrightarrow 2.08 = \ln(x+1) \longrightarrow$ EXPONENTIATE BOTH SIDES, BASE e:

$$e^{2.08} = e^{\ln(x+1)} = x+1 \longrightarrow x = e^{2.08} - 1 \approx 7.005$$

$$-1 \qquad\qquad -1$$

ABOUT 7 UNITS WILL COST \$3556.

4. Find the future value of \$60,000 invested for 8 years at 7%, compounded annually.

THIS IS A SIMPLE INVESTMENT, FUTURE VALUE:

$$S = P\left(1 + \frac{r}{k}\right)^{kt}$$

$P = 60000$, $r = .07$, $t = 8$, $k = 1$ $\longrightarrow$

$$\longrightarrow S = 60000\left(1 + \frac{.07}{1}\right)^{1 \cdot 8} = \$103,091.17$$

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5. The composite SAT scores for entering freshman at GSU are shown in the table below.

Year	1995	1996	1997	1998	1999	2000
SAT score	943	967	973	983	987	1008
Year	2001	2002	2003	2004	2005	2006
SAT score	1028	1052	1056	1080	1098	1104

(a) Using your calculator, find the exponential model which is the best fit for the data. Consider x to be the number of years past 1995 and y to be the SAT score. Report your answer to 3 decimal places.

$$Y = 943.055 \cdot (1.015)^X$$

NOTE: $X = 0$ IN 1995 AND $X = 11$ IN 2006

(b) Use this model to predict the SAT score for this school in 2010.

IN 2010 WE HAVE $X = 15$.

$$Y(15) = 1179$$

(c) According to this model, during what year will the SAT score be above 1300?

USING THE TABLE ON TI-84, THE SAT SCORE IS 1300

FOR $21 < X < 22$, THAT IS DURING YEAR $1995 + 21 = 2016$

ALGEBRA: $Y = 1300 \rightarrow \frac{1300}{943.055} = \frac{943.055(1.015)^X}{943.055} \rightarrow$

$$\frac{1300}{943.055} = 1.015^X \rightarrow X = \log_{1.015}\left(\frac{1300}{943.055}\right) \approx 21.56$$

10

6. (15 points) Find the future value in 18 years of an investment of \$3,000 at a 6.5% annual interest rate in the following cases.

(a) Interest continuously compounded.

FUTURE VALUE OF A STANDARD INVESTEMENT CONTINUOUSLY COMPOUNDED: $S = Pe^{rt}$, WHERE $P = 3000$, $r = .065$, $t = 18$.

HERE $S = 3000\, e^{.065 \cdot 18} \approx \$9665.98$

(b) Interest compounded monthly.

IN THIS PART AND PART (C) WE COMPUTE THE FUTURE VALUE OF A STANDARD INVESTMENT: $S = P\left(1+\frac{r}{k}\right)^{kt}$ FOR $P = 3000$, $r = .065$, $t = 18$

HERE $K = 12 \rightarrow S = 3000\left(1+\frac{.065}{12}\right)^{12 \cdot 18} \approx \$9635.51$

(c) Interest compounded semiannually.

HERE $K = 2$: $S = 3000\left(1+\frac{.065}{2}\right)^{2 \cdot 18} \approx \$9487.75$

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7. (15 points) A couple wants to establish a fund that will provide \$3000 for tuition at the end of each 6-month period for 4 years. If a lump sum can be placed in an account that pays 8% compounded semiannually, what lump sum is required?

"LUMP SUM REQUIRED" IS PRESENT VALUE OF AN ANNUITY:

$$A = \frac{R}{i}\left(1-(1+i)^{-n}\right), \quad R = 3000,\ r = .08,\ K = 2 \text{ (6 MONTHS)}$$

$$t = 4 \to i = \frac{r}{K} = \frac{.08}{2} = .04, \quad n = K \cdot t = 2 \cdot 4 = 8:$$

$$A = \frac{3000}{.04}\left(1-(1+.04)^{-8}\right) \approx 20198.23 \text{ DOLLARS}$$

8. (15 points) MrLawrence invests \$600 at the end of each month in an account that pays 7% compounded monthly. How much will be in the account in 25 years?

"HOW MUCH WILL BE...?" FUTURE VALUE OF A STANDARD ANNUITY:

$$S = \frac{R}{i}\left((1+i)^{n}-1\right), \quad R = 600,\ r = .07,\ K = 12 \text{ (EACH MONTH)}$$

$$t = 25 \to i = \frac{.07}{12}, \quad n = 12 \cdot 25 = 300$$

$$S = \frac{600}{.07/12}\left(\left(1+\frac{.07}{12}\right)^{300}-1\right) \approx \$\,486{,}043.02$$

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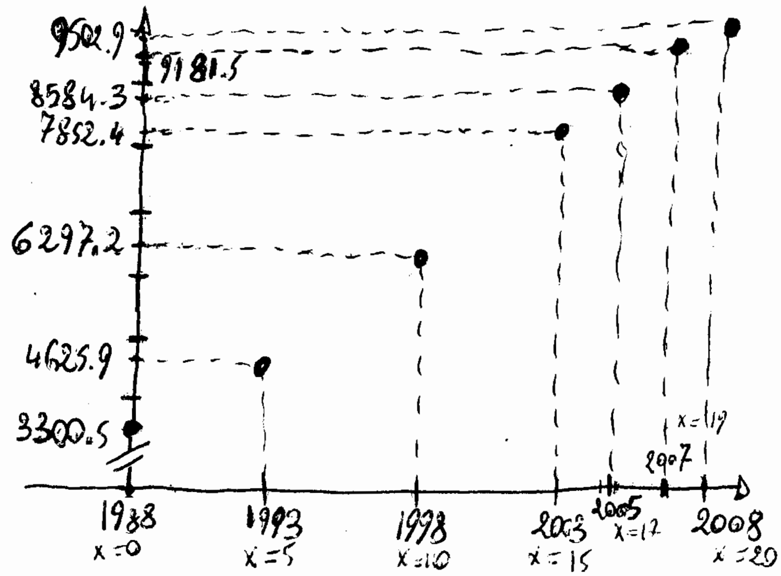
5. The sum of the personal consumption expenditures in the United States, in billions of dollars, for selected years from 1988 to 2008 is show in the following table.

Year	1988	1993	1998	2003	2005	2007	2008
Personal Consumption ($ billions)	3300.5	4625.9	6297.2	7852.4	8584.3	9181.5	9502.9

(a) Make a scatter plot of the data, with x equal to the number of years past 1988 and y equal to the billions of dollars spent. (Clearly report the coordinates of the points)



(b) Using your calculator, find the linear model which is the best fit for the data.

WE CAN SET $X = 0$ FOR 1988, SO THAT $X = 17$ IN 2005; WITH THIS CHOICE WE HAVE THE LINEAR REGRESSION

$Y = 314.7443844\,X + 3182.3832779$

(c) Use the unrounded model to estimate the U.S. personal consumption for 2009.

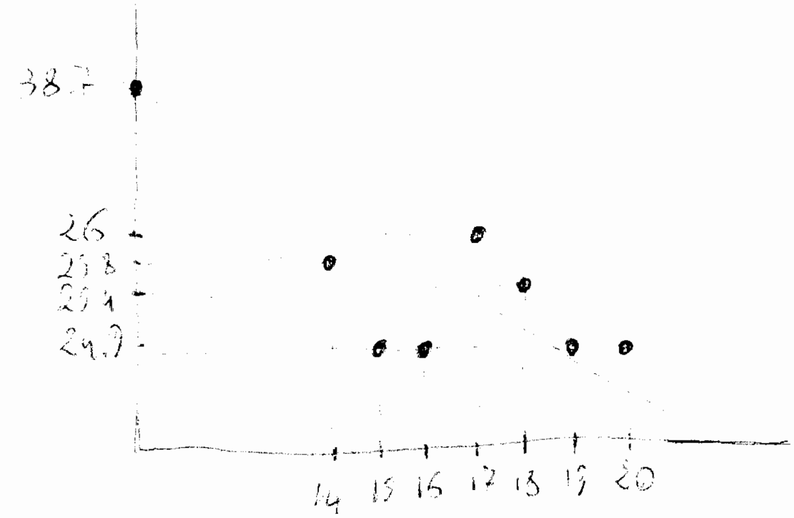
ACCORDING TO THE MODEL IN PART (b), WE PLUG $X = 2009 - 1988 = 21$ TO OBTAIN

$Y = 9792.0154$, THAT IS, THE ESTIMATED US PERSONAL CONSUMPTION FOR 2010 IS OF ABOUT 9.8 TRILLION DOLLARS (9,792,015 MILLION DOLLARS).

5. The table gives the percent of U.S. residents who reported smoking for selected years.

Year	1985	1999	2000	2001	2002	2003	2004	2005
Smoking	38.7	25.8	24.9	24.9	26.0	25.4	24.9	24.9

(a) Make a scatter plot of the data, with x equal to the number of years past 1985 and y the percent of smokers. (Clearly report the coordinates of the points)



(b) Using your calculator, find the linear model which is the best fit for the data.

$Y = -.7197X + 32.6439$

(c) Use the unrounded model to estimate the percentage of U.S. smokers in 2008.

YEAR 2008 IS $X = 2008 - 1985 = 23$

$Y(23) = 21.089$

THE UNROUNDED MODEL ESTIMATES A 21.09 % OF US SMOKERS IN 2008.

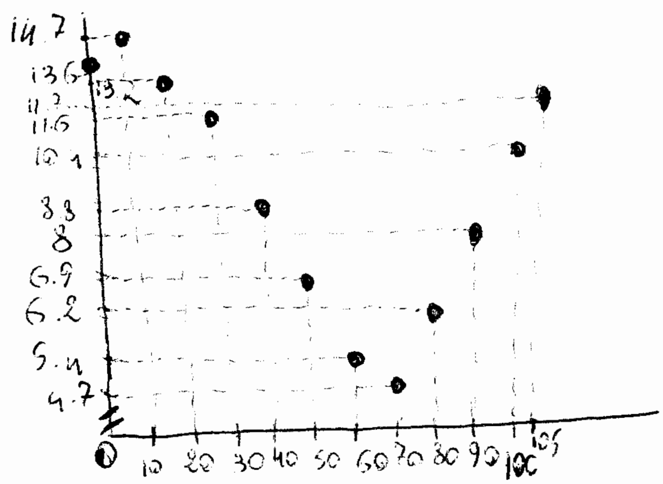
6. The following table gives the percent of U.S. population that is foreign born, for selected years from 1900 to 2005.

Year	1900	1910	1920	1930	1940	1950
Foreign born (%)	13.6	14.7	13.2	11.6	8.8	6.9

Year	1960	1970	1980	1990	2000	2005
Foreign born (%)	5.4	4.7	6.2	8.0	10.4	11.7

(a) Make a scatter plot of the data, with x equal to the number of years past 1900 and y equal to the percent. (Clearly report the coordinates of the points)

WE **MUST** SET X = "# YEARS PAST 1900". FOR EXAMPLE X = 105 IN 200



(b) Using your calculator, find the quadratic model which is the best fit for the data. Report your answer to 4 decimal places.

SAY $y = ax^2 + bx + c$ IS OUR MODEL. $a = .002384$.
IF THE WRONG CHOICE "X = YEAR" IS MADE, THEN b AND C ARE OFF.
$b = -.306578$; $C = 16.491969$. THUS
$y = .0024x^2 - .3066x + 16.492$

(c) Use the function to estimate the percent for 2010.

2010 IS $X = 110$

$Y = P(110) = .0024 \; 110^2 - .3066 \cdot 100 + 16.492 = 11.806$

NOTE THAT THE UNROUNDED ANSWER WOULD BE 11.624 %

THE ESTIMATED PERCENT OF FOREIGN BORN US POPULATION IS 11.806.