Logarithmic and Exponential Problems
(Brief answers follow immediately; details follow that)

PART I

Simplify (without calculator; no decimal answers)

1. $\log 1000$ 2. $\log\sqrt{10}$ 3. $\log_8 2$ 4. $\log_2 8$ 5. $\log_9\left(\frac{1}{81}\right)$ 6. $\log_{0.001} 10$ 7. $\log_{0.2} 125$

8. $3e^{3\ln x}$ 9. $\log_4 \frac{\sqrt[3]{4}}{2}$ 10. $\log_3(\log_3(\log_3 27))$ 11. $\ln e^{\log_2 4}$ 12. $e^{\ln 2-\ln 6}$ 13. $e^{3\ln 1-2\ln 4}$

14. $(2\ln e)^{x^2+2x}$ 15. $\ln\left(2e^{x^2+2x}\right)$ 16. $e^{-\ln 3}$ 17. $e^{5\ln(1/2)}$ 18. $e^{\ln(1/2)+\ln(2/3)}$ 19. $\log_2(\log 10000)$

20. $e^{2+\ln 2}$ 21. $e^{2-\ln 2}$ 22. $\log_{1/4}\frac{16^2}{2^{-3}}$ 23. $\ln\left(\ln\left(\log 10^{\sqrt{e}}\right)\right)$ 24. $10^{\ln e}$ 25. $\frac{1}{t}\ln a^t$

26. $e^{x-\ln x}$ 27. $2^{3\log_2 5}$ 28. $e^{b\ln a}$ 29. $\log_2\frac{\sqrt[3]{16}}{8}$ 30. $\ln\frac{e^{4/3}}{\sqrt{e}}$ 31. $\frac{\ln(a^b)-\ln(a^c)}{b-c}$

32. $\ln\frac{e}{\sqrt[3]{2}}+\ln\sqrt[3]{\frac{2}{e}}$ 33. $(\ln x)(\log_x 3)$ 34. $\ln(3+x)-\ln(x^2-9)$

PART II

Solve for x.

1. $e^{\ln x}=3$ 2. $5^x=25^{x+1}$ 3. $3^{2x+1}=27$ 4. $3^{x^2-3x}=9^{3x-7}$ 5. $3^{2x}3^{x^2}=27$ 6. $e^{2x}e^{5x}=e^{14}$
7. $100^{\sqrt{x}}=10$ 8. $(\sqrt{5})^{x-1}=5^{x+2}$ 9. $9\cdot 2^x=6^x$ 10. $9\cdot 3^x=9^x$ 11. $18^{2x}\cdot 3^{-2x}=6$

12. $2^x+3^x=-1$ 13. $2^x\cdot 4^{-x}=\frac{1}{8}$ 14. $81^{\frac{3x}{5}}=\frac{1}{27}$ 15. $4^{\sqrt{x+1}}=2^{3x-2}$ 16. $2^x\cdot 3^{2x+1}=6^x$

17. $3^{4x}=9^{x-1}$ 18. $2^x\cdot 4^{-x}=\frac{1}{8}$ 19. $5^{2-3x}=25^{-x}$

PART III

Solve for x.

1. $4^{2x-1}=3^{2x+3}$ 2. $3^{2+x}=5^x$ 3. $e^{2x}-e^x=2$ 4. $e^x=3-2e^{-x}$ 5. $4^x+4^{-x}=0$ 6. $\frac{2}{e^x}=\frac{1}{e^{-x}}$

7. $2\cdot 4^x+4^{-x}=\frac{9}{2}$ 8. $\frac{4^x+4^{-x}}{2}=8$ 9. $4(2^x+2^{-x})=17$ 10. $4^{x+1}+4^{1-x}=10$ 11. $3\cdot 4^{2x}+5\cdot 4^x=2$

12. $3^x+8\cdot 3^{-x}=9$ 13. $\frac{4}{2+e^x}=\frac{6}{3+e^{-x}}$ 14. $\frac{3^x-3^{-x}}{3^x+3^{-x}}=9$

PART IV

Solve for x.

1. $\log_x 9=\frac{2}{3}$ 2. $\log x=-1$ 3. $\log_9 x=0.5$ 4. $\log_x 25=-2$ 5. $\log_4 x=2$ 6. $\ln x=0$

7. $2\log x=1$ 8. $\log_x(3-5x)=1$ 9. $\ln\left(\ln e^{-x}\right)=1$ 10. $\log\left(x^2\right)=2$ 11. $(\log x)^2=2$

12. $\log_6\sqrt{x+1}=4$ 13. $\log_9(2x+1)=2$ 14. $\ln(\ln x)=1$ 15. $(\log_5(x+1))^2=4$
16. $\ln\sqrt{x}=\sqrt{\ln x}$ 17. $\log_x(14-5x)=2$ 18. $4\log_4 x=6$ 19. $2\log(3x-2)=4$ 20. $\ln 4e^{-x}=1$
21. $\log_6 6^{4x-1}=19$ 22. $\ln e^{-\ln x}=2$

PART V

Solve for x.

1. $(\log x)^2 - 11\log x + 10 = 0$ 2. $\ln(e^x) - 2\ln e = \ln(e^4)$ 3. $(\log x)^2 + 2\log x = 3$ 4. $2(\log x)^2 + 5\log x = 3$

5. $\ln x = \ln(x-1) + 1$ 6. $\log_4 x = \log_{16}(5x-6)$ 7. $\log_2 x = \log_2(x^2-2)$ 8. $\log x + \log(x-9) = 1$

9. $\log_4 x = \log_8 2x$ 10. $3\log_4(x+2) = 5$ 11. $\log_7(2x+1) = \log_7(3x-8)$ 12. $\log_5(3-x) + \log_5(x+3) = 1$

13. $\log_2 x + \log_2(x+2) = 3$ 14. $\log_3 x + \log_3(x-8) = 2$ 15. $\log_2 x - \log_2(x-7) = 3$

16. $\log_3 x - \log_3(x-3) = 2$ 17. $(\log_2 x)^2 - \log_2 x^2 = 3$ 18. $(\log_2(x-1))^2 = 4\log_2(x-1)$

19. $\log(x+1) - \log(x-1) = 1$ 20. $\log(2x+1) - \log(x-2) = 1$ 21. $\log(x+3) - \log(x-1) = \log x$

22. $\ln x^2 + \ln x^3 + \ln x^5 = 10$ 23. $\ln x^2 + \ln x^3 - \ln x^4 = 1$ 24. $\ln x + 3\ln 2 = \ln\left(\frac{2}{x}\right)$

25. $\ln(4x-2) = \ln 4 - \ln(x-2)$ 26. $\log_2(x+1) + \log_2(3x-5) = \log_2(5x-3) + 2$

PART VI

Miscellaneous

1. Find the domain of (a) $f(x) = \ln\left(\frac{x-1}{x^2-4}\right)$; (b) $f(x) = \ln(3x+5) - \ln(-x)$

2. Find $f^{-1}(t)$ for (a) $f(x) = \frac{e^x - e^{-x}}{2}$; (b) $f(x) = \ln(\ln x)$.

3. Let $f(x) = 2x + 3$ and $g(x) = \ln x$. Find (a) $(f \circ g)(x)$; (b) $(g \circ f)(x)$.

4. Find all x so that $\left|1 - e^{2x}\right| \le 5$.

Log & Exp Answers

(Detailed Answers are on the next page.)

PART I

1. $\mathbf{3}$ 2. $\frac{1}{2}$ 3. $\frac{1}{3}$ 4. $\mathbf{3}$ 5. -2 6. $\frac{-1}{3}$ 7. -3 8. $3x^3$ 9. $\frac{-1}{6}$ 10. $\mathbf{0}$ 11. $\mathbf{2}$

12. $\frac{1}{3}$ 13. $\frac{1}{16}$ 14. 2^{x^2+2x} 15. $x^2+2x+\ln 2$ 16. $\frac{1}{3}$ 17. $\frac{1}{32}$ 18. $\frac{1}{3}$ 19. $\mathbf{2}$

20. $2e^2$ 21. $\frac{e^2}{2}$ 22. $\frac{-11}{2}$ 23. $-\ln 2$ 24. $\mathbf{10}$ 25. $\ln a$ 26. $\frac{e^x}{x}$ 27. $\mathbf{125}$

28. a^b 29. $\frac{-5}{3}$ 30. $\frac{5}{6}$ 31. $\ln a$ 32. $\frac{2}{3}$ 33. $\ln 3$ 34. $-\ln(x-3)$

PART II

1. $\mathbf{3}$ 2. -2 3. $\mathbf{1}$ 4. $2,7$ 5. $-3,1$ 6. $\mathbf{2}$ 7. $\frac{1}{4}$ 8. -5 9. $\mathbf{2}$ 10. $\mathbf{2}$ 11. $\frac{1}{2}$

12. $\varnothing$ 13. $\mathbf{3}$ 14. $\frac{-5}{4}$ 15. $\frac{16}{9}$ 16. -1 17. -1 18. $\mathbf{3}$ 19. $\mathbf{2}$

PART III

1. $\frac{3\ln 3+\ln 4}{2(\ln 4-\ln 3)}$ 2. $\frac{2\ln 3}{\ln 5-\ln 3}$ 3. $\ln 2$ 4. $0,\ln 2$ 5. $\varnothing$ 6. $\ln\sqrt{2}$ 7. $-1,\frac{1}{2}$

8. $\log_4(8\pm 3\sqrt{7})$ 9. ± 2 10. $\pm\frac{1}{2}$ 11. $-\log_4 3$ 12. $0,\log_3 8$ 13. $\frac{1}{2}\ln\left(\frac{2}{3}\right)$ 14. $\varnothing$

PART IV

1. $\mathbf{27}$ 2. $\frac{1}{10}$ 3. $\mathbf{3}$ 4. $\frac{1}{5}$ 5. $\mathbf{16}$ 6. $\mathbf{1}$ 7. $\sqrt{10}$ 8. $\frac{1}{2}$ 9. $-e$ 10. ± 10

11. $10^{\pm\sqrt{2}}$ 12. 6^8-1 13. $\mathbf{40}$ 14. e^e 15. $\frac{-24}{25},24$ 16. $1,e^4$ 17. $\mathbf{2}$ 18. $\mathbf{8}$

19. $\mathbf{34}$ 20. $-1+\ln 4$ 21. $\mathbf{5}$ 22. e^{-2}

PART V

1. $10,10^{10}$ 2. $\mathbf{6}$ 3. $10^{-3},10x$ 4. $10^{-3},\sqrt{10}$ 5. $\frac{e}{e-1}$ 6. $2,3$ 7. $\mathbf{2}$ 8. $\mathbf{10}$

9. $\mathbf{4}$ 10. $4^{\frac{5}{3}}-2$ 11. $\mathbf{9}$ 12. ± 2 13. $\mathbf{2}$ 14. $\mathbf{9}$ 15. $\mathbf{8}$ 16. $\frac{27}{8}$ 17. $\frac{1}{2},8$

18. $2,17$ 19. $\frac{11}{9}$ 20. $\frac{21}{8}$ 21. $\mathbf{3}$ 22. e 23. e 24. $\frac{1}{2}$ 25. $\frac{5}{2}$ 26. $\mathbf{7}$

PART VI

1.(a) $(-2,1)\cup(2,\infty)$ 1.(b) $\left(\frac{-5}{3},0\right)$ 2.(a) $f^{-1}(t)=\ln\left(t+\sqrt{t^2+1}\right)$ 2.(b) $f^{-1}(t)=e^{\left(e^t\right)}$

3.(a) $3+2\ln x$ 3.(b) $\ln(2x+3)$ 4. $(-\infty,\ln\sqrt{6}]$

Detailed Answers

Part I

Almost all of the problems in this part make use of the **Fundamental Duality** that exists between the log function and the exponential function:

$$\log_a u = v \Leftrightarrow a^v = u$$

For the log function on the left, the quantity u is the argument of the log function and the quantity v is its value. These roles get reversed for the exponential function on the right: v is now the argument of the exponential function and u is its value.

So, for Problem #1, if we set $\log 1000$ equal to v (i.e., $\log 1000 = v$), we have by the **duality** $10^v = 1000$. At this point we hope we can write **1000** as **10** to some power so we can use the **One-to-One** rule:

$$a^s = a^t \Leftrightarrow s = t$$

Since $1000 = 10^3$, we have $10^v = 10^3$, so $v = 3$.

Problem #3 is a variation on this theme. Setting $\log_8 2 = v$, we get from the **duality** $8^v = 2$. Now instead of trying to write **2** as a power of **8**, we recognize that $8 = 2^3$, so our dual statement becomes $(2^3)^v = 2$. Using laws of exponents, we get $2^{3v} = 2^1$, so by the **One-to-One** rule we have $3v = 1$ and hence $v = \frac{1}{3}$.

Problem #8 introduces the use of two sets of rules: the Log Laws and the Identity Laws.

Log Laws:

(LL1) $\log_a (AB) = \log_a A + \log_a B$

(LL2) $\log_a \left(\frac{A}{B}\right) = \log_a A - \log_a B$

(LL3) $\log_a (A^r) = r \cdot \log_a A$

Identity Laws:

(Id1) $\log_a (a^t) = t$

(Id2) $a^{\log_a t} = t$

The Identity Laws can be written in English which is sometimes useful if expressions get messy looking:

(Id1) **log** base a of (a raised to any power) is just the "power".
(Id2) a raised to the power (**log** base a of anything) is just the "thing".

To start Problem #8, we first use LL3 to write $3\ln x$ as $\ln(x^3)$. Then the problem becomes $3e^{\ln(x^3)}$. But, by Id2, since $\ln$ means $\log_e$, this becomes $3x^3$. (e to the power (**ln** of anything) is just the "thing".)

Problem #11 uses Id1. Here, $\ln e^{\log_2 4}$ is just $\log_2 4$. (**ln** of (e raised to any power) is just the "power".) But, we're not done, by the **fundamental duality** $\log_2 4 = 2$ since $2^2 = 4$.

Problems like #10 and #23 are attacked by the Order of Operations – work inside to outside. For Problem #23 one starts way inside with $\log 10^{\sqrt{e}}$ which, by Id1 is just $\sqrt{e}$ which we write as $e^{\frac{1}{2}}$ for the next part. Working our way outside, we now have $\ln(e^{\frac{1}{2}})$, which by Id1 again is $\frac{1}{2}$. This leaves us with $\ln\frac{1}{2}$. This is technically a correct answer, but it looks ugly. We would rather express $\frac{1}{2}$ as 2^{-1} and use LL3:

$\ln 2^{-1} = (-1)\ln 2 = -\ln 2$.

Detailed Answers (page 2)

Problems like #20, #21, and #26 make use of the Laws of Exponents first. In Problem #26 we have $e^{x-\ln x} = \dfrac{e^x}{e^{\ln x}}$. Here we've used the "dividing the bases means subtracting the exponents" rule backwards: "subtracting the exponents means dividing the bases." Now Id2 is used to write $e^{\ln x}$ as just x. So the answer is $\dfrac{e^x}{x}$.

Problem #31 is interesting because it involves the distributive law. To begin with, we can use LL3 to write the numerator as $b\ln(a) - c\ln(a)$. Since $\ln(a)$ is a common factor of the two terms, we can distribute it out: $(b-c)\ln(a)$. So our fraction becomes $\dfrac{(b-c)\ln a}{b-c}$, and the quantity $b-c$ can be cancelled away, leaving just $\ln a$.

For Problem #33 we need the alternative form of the **Change of Base** formula:

Usual form: $\log_a b = \dfrac{\log_c b}{\log_c a}$; Alternative form: $\log_a b = (\log_a c)(\log_c b)$

In the Usual form, a log can be replaced by a quotient of logs, each having a new common base. In the Alternative form, a log can be replaced by a product of logs, where the argument of the first is the base of the second.

So in #33, the argument of the first log (i.e., x) is the base of the second, so this is simply $\ln 3$.

Part II

All of the problems here (except #12) make use of the **One-to-One** rule (see above). Simply express each side of the equation as a common number raised a power, and then equate the powers. For example, Problem #5 can be re-written as $3^{2x+x^2} = 3^3$. Thus, we equate the exponents and solve the resulting quadratic equation: $2x + x^2 = 3$. Thus x equals -3 or 1.

Problems like #9, #11, and #16 are interesting because they require a deeper understanding. For example, it should be easy to see that Problem #11 can be re-written as $2^{2x} \cdot 3^{2x} = 2^1 \cdot 3^1$ after breaking apart the **18** and using laws of exponents on the **3**'s. Now the One-to-One rule still applies for each base, and here we must have $2x = 1$ in each case, so $x = \dfrac{1}{2}$.

Problem #12 also requires a deeper understanding. We know that an exponential can never be **0** or negative. So both 2^x and 3^x are positive. Can two positive numbers add up to **negative 1**? No, way. Hence there is no solution to this problem.

Part III

Problems #1 and #2 are exponential equations to which the One-to-One rule does not apply since the bases cannot be easily made the same. In this type of problem the technique is to take logs of both sides and use LL3. Actually, any base log will work, but typically we use **ln**. For example in Problem #1, we take **ln** of both sides which yields: $\ln(4^{2x-1}) = \ln(3^{2x+3})$. Using LL3 gives $(2x-1)\ln 4 = (2x+3)\ln 3$. This is now just a linear equation in x which you should be able to do in your sleep by this time – as long as you remember that the factors $\ln 4$ and $\ln 3$ are just numbers and are treated just like you would treat the **5** in something like $(4x-3) \cdot 5$, i.e., you would distribute it: $20x - 15$. So, here we distribute the two **ln**'s giving:

Detailed Answers (page 3)

$(2\ln 4)x - \ln 4 = (2\ln 3)x + 3\ln 3$. Combining like terms and moving constants to the right gives: $(2\ln 4 - 2\ln 3)x = 3\ln 3 + \ln 4$. Thus $x = \dfrac{3\ln 3 + \ln 4}{2\ln 4 - 2\ln 3}$.

The rest of the problems in this part involve (eventually) solving an **equation of quadratic type** (or an **equation of quadratic form**) which you saw in Basic Algebra. Remember, these were things you saw like $x^{\frac{2}{3}} - x^{\frac{1}{3}} - 6 = 0$, where you treated $x^{\frac{2}{3}}$ as a quadratic in the quantity $x^{\frac{1}{3}}$, that is, you treated $x^{\frac{2}{3}}$ as $\left(x^{\frac{1}{3}}\right)^2$.

Then the equation becomes $\left(x^{\frac{1}{3}}\right)^2 - \left(x^{\frac{1}{3}}\right) - 6 = 0$. Typically you are taught to now make a substitution, something like, let $u = x^{\frac{1}{3}}$, so the equation becomes $u^2 - u - 6 = 0$ which you can solve quickly to get $u = -2$ or $u = 3$. Then you replace u by its x equivalent: $x^{\frac{1}{3}} = -2$ and $x^{\frac{1}{3}} = 3$. This yields $x = -8$ or $x = 27$ as solutions to the original problem.

However, for most of the problem here, you have to do a little algebra first to get one of these equations of quadratic form. And the algebra involved often includes using laws of exponents, removing fractions and eliminating a negative exponent. So for example in Problem #7 we begin by removing the fractions: $4 \bullet 4^x + 2 \bullet 4^{-x} = 9$. Now remove the negative exponent: $4 \bullet 4^x + \dfrac{2}{4^x} = 9$. And again remove the fraction: $4 \bullet (4^x)^2 + 2 = 9 \bullet 4^x$. This is our equation of quadratic form. Setting $u = 4^x$ we get $4u^2 - 9u + 2 = 0$, so $u = \dfrac{1}{4}$ of $u = 2$. Thus, $4^x = 2$ or $4^x = \dfrac{1}{4}$, so $x = \dfrac{1}{2}$ or $x = -1$.

Caution: Do some deeper thinking about Problem #5 – there is no work to solving this one.

Part IV

For the most part, these problems simply use the **Fundamental Duality**. For example in Problem #8 the dual statement is $x^1 = 3 - 5x$. Thus, $x = \dfrac{1}{2}$. **Remember, however, in log equations none of the proposed solutions can make the base 0, 1 or negative, and none can make any argument negative.** In this case our proposed solution does not violate either of those rules, so we are done. (Had either of the rules been violated, we would have to discard our "solution.")

Some of the problems (like #9, #20-22) use one of the Identity Laws.

Problem #16 is of importance to distinguish between the "log of (something to a power)" versus the "(log of something) to a power." To see the difference here, let's re-write both sides using fractional exponents: $\ln\left(x^{\frac{1}{2}}\right) = (\ln x)^{\frac{1}{2}}$. The left hand side simplifies by LL3, but there is no log rule to simplify the right hand side.

We are then left with: $\left(\dfrac{1}{2}\right)\ln x = \sqrt{\ln x}$ which is a **radical equation**. Since a radical is already isolated, we square both sides: $\left(\dfrac{1}{4}\right)(\ln x)^2 = \ln x$ which is an equation in quadratic form. Setting $u = \ln x$, we have

Detailed Answers (page 4)

$\left(\frac{1}{4}\right)u^2 = u$, which leads to $u^2 - 4u = 0$ and hence $u = 0$ or $u = 4$. But this means $\ln x = 0$ or $\ln x = 4$. Hence, by our duality: $x = 1$ or $x = e^4$

Part V

This part starts with log equations that are really equations of quadratic form. For example, in Problem #3 by setting $u = \log x$, we have $u^2 + 2u - 3 = 0$. Thus, $u = -3$ or $u = 1$, so $\log x = -3$ or $\log x = 1$ which means by the **Fundamental Duality** that $x = e^{-3}$ or $x = e$.

For many of the other problems the **Log Laws** play a prominent role. The idea is to create one of two types of equations.

(1) $\log_a s = \log_a t$ which forces $s = t$ by a **One-to-One** law for logs.

(2) $\log_a u = v$ which forces $a^v = u$ by the **Fundamental Duality**

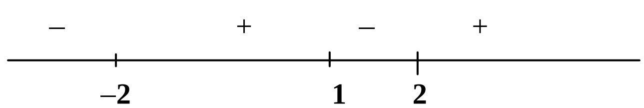
Problem #7 is already in the first form, so $x = x^2 - 2$. This leads to $x = -1$ or $x = 2$. However, since there is a $\log_2 x$ in the original problem and $x = -1$ causes its argument to be negative, we must discard this solution, which leaves only $x = 2$. **NOTE:** we did not discard $x = -1$ because it was negative, but because it made one (actually both) of the arguments negative.

Applying LL2 to Problem #15 produces $\log_2\left(\frac{x}{x-7}\right) = 3$. Which by the **Duality** means $\frac{x}{x-7} = 2^3 = 8$. Thus $x = 8$.

Don't let Problem #6 throw you for a loop. Since the bases of the two logs are not the same, we must first use one of the **Change of Base** formulas to create a common base. How about changing base **4** into base **16**: $\log_4 x = (\log_4 16)(\log_{16} x) = 2\log_{16} x = \log_{16}(x^2)$. So our equation becomes $\log_{16}(x^2) = \log_{16}(5x-6)$ which means by the **One-to-One** law that $x^2 = 5x - 6$ which is easily solved.

Part VI

For Problem #1a we know that the argument of the log function must be positive. This means $\frac{x-1}{x^2-4} > 0$ which is a non-linear inequality. Factoring and doing plus/minus analysis with critical points **1** and ± 2, we see that



So the domain is $(-2,1) \cup (2,\infty)$.

For Problem #1b we take the intersection of the two domains: $\left(\frac{-5}{3},\infty\right) \cap (-\infty,0)$ which equals $\left(\frac{-5}{3},0\right)$.

Detailed Answers (page 5)

Problem 2 is a bit tricky since it is asking for $f^{-1}(t)$, not $f^{-1}(y)$ or $f^{-1}(x)$. It is a minor point, that seems like a trick to most students, but the concept here of a "dummy variable" is important to recognize. To start Problem #2a we set $f(x)$ equal to t and solve for x which will end up involving an equation in quadratic form.

$\frac{e^x - e^{-x}}{2} = t \Rightarrow e^x - \frac{1}{e^x} = 2t \Rightarrow (e^x)^2 - 2te^x - 1 = 0$. Setting $u = e^x$ leads to a quadratic equation that does not factor, so we use the quadratic formula to get $u = t \pm \sqrt{t^2+1} = e^x$. However, we notice (some deep thinking needed here!!!) that $\sqrt{t^2+1}$ is bigger than t, so the quantity $t - \sqrt{t^2+1}$ is negative and hence cannot be a value for e^x. So, after discarding that value, we have $e^x = t + \sqrt{t^2+1}$ which means $x = f^{-1}(t) = t + \sqrt{t^2+1}$.

We attack Problem #2b in a similar manner: setting $\ln(\ln x) = t$ and using the **duality** twice gives first $\ln x = e^t$, then $x = e^{(e^t)}$ which is the required inverse.

Problem #4 is an "inside" absolute value inequality which leads to the double inequality: $-5 \le 1 - e^{2x} \le 5$. Subtract 1 from all three pieces: $-6 \le -e^{2x} \le 4$. Multiply through by -1: $6 \ge e^{2x} \ge -4$. But now we know that e^{2x} must be positive, so we have $0 < e^{2x} \le 6$. Now since the square root function and the ln function are both increasing functions we get first $0 < e^x \le \sqrt{6}$, and then $-\infty < x \le \ln\sqrt{6}$. Hence the answer is $(-\infty, \sqrt{6}]$.