PreCalculus Formulas

Sequences and Series:

Binomial Theorem	Arithmetic Last Term	Geometric Last Term
$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$	$a_n = a_1 + (n-1)d$	$a_n = a_1 r^{n-1}$
Find the r^{th} term	Arithmetic Partial Sum	Geometric Partial Sum
$\binom{n}{r-1} a^{n-(r-1)} b^{r-1}$	$S_n = n\left(\frac{a_1 + a_n}{2}\right)$	$S_n = a_1\left(\frac{1-r^n}{1-r}\right)$

Functions:

To find the inverse function: $f^{-1}(x)$

1. Set function $= y$
2. Interchange the variables
3. Solve for y

Composition of functions:

$(f \circ g)(x) = f(g(x))$

$(g \circ f)(x) = g(f(x))$

$(f \circ f^{-1})(x) = x$

Algebra of functions: $(f+g)(x) = f(x) + g(x)$; $(f-g)(x) = f(x) - g(x)$

$(f \bullet g)(x) = f(x) \bullet g(x)$; $(f/g)(x) = f(x)/g(x)$, $g(x) \neq 0$

Domains:: $D(f(x)) \cap D(g(x))$

Domain (usable x's)
Watch for problems with zero denominators and with negatives under radicals.

Range (y's used)

Difference Quotient

$$\frac{f(x+h) - f(x)}{h}$$

terms not containing a mult. of h will be eliminated.

Asymptotes: (vertical)
Check to see if the denominator could ever be zero.

$$f(x) = \frac{x}{x^2 + x - 6}$$

Vertical asymptotes at $x = -3$ and $x = 2$

Asymptotes: (horizontal)

1. $f(x) = \frac{x+3}{x^2 - 2}$

top power < bottom power means $y = 0$ (x-axis)

2. $f(x) = \frac{4x^2 - 5}{3x^2 + 4x + 6}$

top power = bottom power means $y = 4/3$ (coefficients)

3. $f(x) = \frac{x^3}{x+4}$ None!

top power > bottom power

Complex and Polars:

DeMoivre's Theorem:

$$[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\bullet\theta + i\sin n\bullet\theta)$$

$r = \sqrt{a^2 + b^2}$ $\quad x = r\cos\theta$

$\theta = \arctan\frac{b}{a}$ $\quad y = r\sin\theta$

$(r, \theta) \rightarrow (x, y)$

$a + bi$

$i = \sqrt{-1}$

$i^2 = -1$

Determinants:

$$\begin{vmatrix} 3 & 5 \\ 4 & 3 \end{vmatrix} = 3\bullet3 - 5\bullet4$$

Use your calculator for 3x3 determinants.

Cramer's Rule:

$ax + by = c$

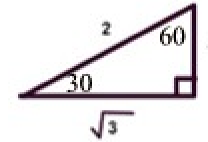
$dx + ey = f$

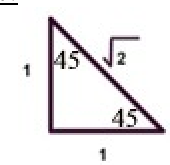
$$\frac{1}{\begin{vmatrix} a & b \\ d & e \end{vmatrix}}\left(\begin{vmatrix} c & b \\ f & e \end{vmatrix}, \begin{vmatrix} a & c \\ d & f \end{vmatrix}\right)$$

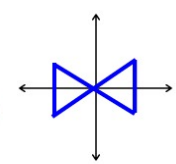
Also apply Cramer's rule to 3 equations with 3 unknowns.

Trig:

Reference Triangles:







$\sin\theta = \frac{o}{h}$; $\cos\theta = \frac{a}{h}$; $\tan\theta = \frac{o}{a}$

$\csc\theta = \frac{h}{o}$; $\sec\theta = \frac{h}{a}$; $\cot\theta = \frac{a}{o}$

BowTie

Positioning of reference triangles.

Analytic Geometry:

<u>Circle</u> $(x-h)^2+(y-k)^2=r^2$ Remember "completing the square" process for all conics.	<u>Ellipse</u> $\dfrac{(x-h)^2}{a^2}+\dfrac{(y-k)^2}{b^2}=1$ larger denominator → major axis and smaller denominator → minor axis	c → focus length where major length is hypotenuse of right triangle. Latus rectum lengths from focus are b^2/a	<u>Eccentricity:</u> $e=0$ circle $0<e<1$ ellipse $e=1$ parabola $e>1$ hyperbola
<u>Parabola</u> $(x-h)^2=4a(y-k)$ $(y-k)^2=4a(x-h)$ vertex to focus = a, length to directrix = a, latus rectum length from focus = $2a$	<u>Hyperbola</u> $\dfrac{(x-h)^2}{a^2}-\dfrac{(y-k)^2}{b^2}=1$ Latus length from focus b^2/a	a→transverse axis b→conjugate axis c→focus where c is the hypotenuse. asymptotes needed	

Induction:

Find $P(1)$:
Assume $P(k)$ is true:
Show $P(k+1)$ is true:

Rate of Growth/Decay: $y=y_0e^{kt}$

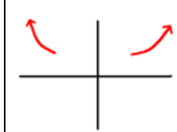
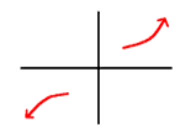
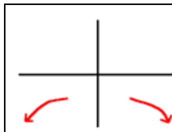
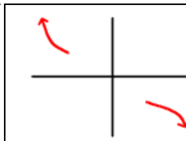
y = end result, y_0 = start amount,
Be sure to find the value of k first.

Polynomials:

<u>Remainder Theorem:</u> Substitute into the expression to find the remainder. [(x + 3) substitutes -3]	<u>Synthetic Division Mantra:</u> "Bring down, multiply and add, multiply and add…" [when dividing by (x - 5), use +5 for synthetic division]	<u>Depress equation</u> $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ (also use calculator to examine roots)
<u>Descartes' Rule of Signs</u> 1. Maximum possible # of positive roots → number of sign changes in $f(x)$ 2. Maximum possible # of negative roots → number of sign changes in $f(-x)$	<u>Analysis of Roots</u> P N C Chart * all rows add to the degree * complex roots come in conjugate pairs * product of roots - sign of constant (same if degree even, opposite if degree odd) * decrease P or N entries by 2	<u>Upper bounds</u>: All values in chart are + <u>Lower bounds</u>: Values alternate signs <u>No remainder:</u> Root <u>Sum of roots</u> is the coefficient of second term with sign changed. <u>Product of roots</u> is the constant term (sign changed if odd degree, unchanged if even degree).

<u>Far-left/Far-right Behavior of a Polynomial</u>

The leading term (a_nx^n) of the polynomial determines the far-left/far-right behavior of the graph according to the following chart. ("Parity" of n → whether n is odd or even.)

<table>
<tr><td colspan="2" rowspan="2">a_nx^n</td><td colspan="2">LEFT-HAND BEHAVIOR</td></tr>
<tr><td>n is even
(same as right)</td><td>n is odd
(opposite right)</td></tr>
<tr><td rowspan="2">RIGHT-HAND BEHAVIOR or
Leading Coefficient Test</td><td>$a_n > 0$</td><td>
always positive</td><td>
negative $x<0$
positive $x>0$</td></tr>
<tr><td>$a_n < 0$</td><td>
always negative</td><td>
positive $x<0$
negative $x>0$</td></tr>
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